1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I..Numeral : In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called ***digits*** to represent any number.

A group of digits, denoting a number is called a ***numeral.***

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores (10^7)	Ten Lacs (Millions) (10^6)	Lacs (10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	**8**	**9**	**7**	**4**	**5**	**1**	**3**	**2**

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II Place Value or Local Value of a Digit in a Numeral :

In the above numeral :

Place value of 2 is (2 x 1) = 2; Place value of 3 is (3 x 10) = 30;

Place value of 1 is (1 x 100) = 100 and so on.

Place value of 6 is $6 \times 10^8 = 600000000$

III.Face Value : The ***face value*** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV.TYPES OF NUMBERS

1.Natural Numbers : Counting numbers 1, 2, 3, 4, 5,..... are called ***natural*** *numbers.*

2.Whole Numbers : All counting numbers together with zero form the set of ***whole numbers***. Thus,

(i) 0 is the only whole number which is not a natural number.

(ii) Every natural number is a whole number.

3.Integers : All natural numbers, 0 and negatives of counting numbers *i.e.,* {…, -3, -2, -1, 0, 1, 2, 3,…..} together form the set of integers.

(i) **Positive Integers** : {1, 2, 3, 4, …..} is the set of all positive integers.

(ii) **Negative Integers** : {- 1, - 2, - 3,…..} is the set of all negative integers.

(iii) **Non-Positive and Non-Negative Integers** : 0 is neither positive nor negative. So, {0, 1, 2, 3,….} represents the set of non-negative integers, while {0, -1, -2, -3, …..} represents the set of non-positive integers.

4. Even Numbers : A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8, 10, etc.

5. Odd Numbers : A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7, 9, 11, etc.

6. Prime Numbers : A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :

Find a whole number nearly greater than the square root of p. Let k > *jp. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.

e.g,,We have to find whether 191 is a prime number or not. Now, 14 > V191.

Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.

191 is not divisible by any of them. So, 191 is a prime number.

7.Composite Numbers : Numbers greater than 1 which are not prime, are known as composite numbers, e.g., 4, 6, 8, 9, 10, 12.

Note : (i) 1 is neither prime nor composite.

(ii) 2 is the only even number which is prime.

(iii) There are 25 prime numbers between 1 and 100.

8. Co-primes : Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g., (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes,

V.TESTS OF DIVISIBILITY

1. Divisibility By 2 : A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.

Ex. 84932 is divisible by 2, while 65935 is not.

2. Divisibility By 3 : A number is divisible by 3, if the sum of its digits is divisible by 3.

Ex.592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.

But, 864329 is not divisible by 3, since sum of its digits =(8 + 6 + 4 + 3 + 2 + 9) = 32, which is not divisible by 3.

3. Divisibility By 4 : A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.

But, 749282 is not divisible by 4, since the number formed by the last tv/o digits is 82, which is not divisible by 4.

4. Divisibility By 5 : A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.

5. Divisibility By 6 : A number is divisible by 6, if it is divisible by both 2 and 3. Ex. The number 35256 is clearly divisible by 2.

Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3. Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

6. Divisibility By 8 : A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.

But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

7. Divisibility By 9 : A number is divisible by 9, if the sum of its digits is divisible by 9.

Ex. 60732 is divisible by 9, since sum of digits * (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9.

But, 68956 is not divisible by 9, since sum of digits = (6 + 8 + 9 + 5 + 6) = 34, which is not divisible by 9.

8. Divisibility By 10 : A number is divisible by 10, if it ends with 0.

Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. Divisibility By 11 : A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

Ex. The number 4832718 is divisible by 11, since :

(sum of digits at odd places) - (sum of digits at even places)

(8 + 7 + 3 + 4) - (1 + 2 + 8) = 11, which is divisible by 11.

10. Divisibility By 12 ; A number is divisible by 12, if it is divisible by both 4 and 3.

Ex. Consider the number 34632.

(i) The number formed by last two digits is 32, which is divisible by 4,

(ii) Sum of digits = (3 + 4 + 6 + 3 + 2) = 18, which is divisible by 3. Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. Divisibility By 14 : A number is divisible by 14, if it is divisible by 2 as well as 7.

12. Divisibility By 15 : A number is divisible by 15, if it is divisible by both 3 and 5.

13. Divisibility By 16 : A number is divisible by 16, if the number formed by the last4 digits is divisible by 16.

Ex.7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. Divisibility By 24 : A given number is divisible by 24, if it is divisible by both3 and 8.

15. Divisibility By 40 : A given number is divisible by 40, if it is divisible by both 5 and 8.

16. Divisibility By 80 : A given number is divisible by 80, if it is divisible by both 5 and 16.

Note : If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.

If p arid q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

Ex. 36 is divisible by both 4 and 6, but it is not divisible by (4x6) = 24, since 4 and 6 are not co-primes.

VI MULTIPLICATION BY SHORT CUT METHODS

1. Multiplication By Distributive Law :

(i) a x (b + c) = a x b + a x c (ii) ax(b-c) = a x b-a x c.

Ex. (i) 567958 x 99999 = 567958 x (100000 - 1)

= 567958 x 100000 - 567958 x 1 = (56795800000 - 567958) = 56795232042. (ii) 978 x 184 + 978 x 816 = 978 x (184 + 816) = 978 x 1000 = 978000.

2. Multiplication of a Number By 5^n : Put n zeros to the right of the multiplicand and divide the number so formed by 2^n

Ex. 975436 x 625 = 975436 x 5^4= 9754360000 = 609647600

VII. BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$ 2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$ 4. $(a + b)^2 + (a - b)^2 = 2 (a^2 + b^2)$
5. $(a^2 - b^2) = (a + b) (a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$
7. $(a^3 + b^3) = (a +b) (a^2 - ab + b^2)$ 8. $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 -3abc) = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

Dividend = (Divisor x Quotient) + Remainder

IX. {i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.
(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.
(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a progression.

1. Arithmetic Progression (A.P.) : If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference* of the A.P.

An A.P. with first term a and common difference d is given by a, (a + d), (a + 2d),(a + 3d),.....

The **nth term of this A.P. is given by T_n =a (n - 1) d.**

The sum of n terms of this A.P.

$S_n = n/2 [2a + (n - 1) d] = n/2$ (first term + last term).

SOME IMPORTANT RESULTS :

(i) $(1 + 2 + 3 +\dots + n) =n(n+1)/2$
(ii) $(l^2 + 2^2 + 3^2 + ... + n^2) = n (n+1)(2n+1)/6$
(iii) $(1^3 + 2^3 + 3^3 + ... + n^3) =n^2(n+1)^2$

2. Geometrical Progression (G.P.) : A progression of numbers in which every term bears a constant ratio with its preceding term, is called a geometrical progression.

The constant ratio is called the common ratio of the G.P. A G.P. with first term a and common ratio r is :

a, ar, ar^2,

In this G.P. $T_n = ar^{n-1}$

sum of the n terms, Sn= $\dfrac{a(1-r^n)}{(1-r)}$

SOLVED EXAMPLES

Ex. 1. Simplify : (i) 8888 + 888 + 88 + 8
(ii) 11992 - 7823 - 456

Sol. i) 8888 ii) 11992 - 7823 - 456 = 11992 - (7823 + 456)
= 11992 - 8279 = 3713-

$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 9872 \\ \hline \end{array} \qquad \begin{array}{r} 7823 \\ +\ 456 \\ \hline 8279 \\ \hline \end{array} \qquad \begin{array}{r} 11992 \\ -\ 8279 \\ \hline 3713 \\ \hline \end{array}$$

Ex. 2, What value will replace the question mark in each of the following equations ?
(i) ? - 1936248 = 1635773 (ii) 8597 - ? = 7429 - 4358

Sol. (i) Let x - 1936248=1635773.Then, x = 1635773 + 1936248=3572021.
(ii) Let 8597 - x = 7429 - 4358.
Then, x = (8597 + 4358) - 7429 = 12955 - 7429 = 5526.

Ex. 3. What could be the maximum value of Q in the following equation? 5P9 + 3R7 + 2Q8 = 1114

Sol. We may analyse the given equation as shown :
Clearly, 2 + P + R + Q = ll.
So, the maximum value of Q can be
(11 - 2) i.e., 9 (when P = 0, R = 0);

$$\begin{array}{r} 1\ \ 2\quad \\ 5\ \ P\ \ 9 \\ 3\ \ R\ \ 7 \\ 2\ \ Q\ \ 8 \\ \hline 11\ \ 1\ \ 4 \\ \hline \end{array}$$

Ex. 4. Simplify : (i) 5793405 x 9999 (ii) 839478 x 625
Sol.
i)5793405x9999=5793405(10000-1)=57934050000-5793405=57928256595.b

ii) 839478 x 625 = 839478 x 5^4 = $\frac{8394780000}{16}$ = 524673750.

Ex. 5. Evaluate : (i) 986 x 237 + 986 x 863 (ii) 983 x 207 - 983 x 107
Sol.
(i) 986 x 137 + 986 x 863 = 986 x (137 + 863) = 986 x 1000 = 986000.
(ii) 983 x 207 - 983 x 107 = 983 x (207 - 107) = 983 x 100 = 98300.

Ex. 6. Simplify : (i) 1605 x 1605 ii) 1398 x 1398
Sol.
i) 1605 x 1605 = $(1605)^2$ = $(1600 + 5)^2$ = $(1600)^2 + (5)^2$ + 2 x 1600 x 5
= 2560000 + 25 + 16000 = 2576025.
(ii) 1398 x 1398 - $(1398)^2$ = $(1400 - 2)^2$= $(1400)^2 + (2)^2$ - 2 x 1400 x 2

$=1960000 + 4 - 5600 = 1954404.$

Ex. 7. Evaluate : (313 x 313 + 287 x 287).
Sol.
$(a^2 + b^2) = 1/2 [(a + b)^2 + (a- b)^2]$
$(313)^2 + (287)^2 = 1/2 [(313 + 287)^2 + (313 - 287)^2] = ½[(600)^2 + (26)^2]$
$= 1/2 (360000 + 676) = 180338.$

Ex. 8. Which of the following are prime numbers ?
(i) 241 (ii) 337 (Hi) 391 (iv) 571
Sol.
(i) Clearly, 16 > Ö241. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.
241 is not divisible by any one of them.
241 is a prime number.

(ii) Clearly, 19>Ö337. Prime numbers less than 19 are 2, 3, 5, 7, 11,13,17.
337 is not divisible by any one of them.
337 is a prime number.
(iii) Clearly, 20 > Ö39l". Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
We find that 391 is divisible by 17.
391 is not prime.
(iv) Clearly, 24 > Ö57T. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.
571 is not divisible by any one of them.
571 is a prime number.

Ex. 9. Find the unit's digit in the product (2467)163 x (341)72.
Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.
Now, 74 gives unit digit 1.
7^{152} gives unit digit 1,
$\therefore 7^{153}$ gives unit digit (l x 7) = 7. Also, 1^{72} gives unit digit 1.
Hence, unit's digit in the product = (7 x 1) = 7.

Ex. 10. Find the unit's digit in $(264)^{102} + (264)^{103}$
Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103.}$
Now, 4^2 gives unit digit 6.
$\therefore (4)^{102}$ gives unjt digit 6.
$\therefore$ (4)103 gives unit digit of the product (6 x 4) i.e., 4.
Hence, unit's digit in (264)m + (264)103 = unit's digit in (6 + 4) = 0.

Ex. 11. Find the total number of prime factors in the expression $(4)^{11} \times (7)^5 \times (11)^2$.
Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{11} \times 2^{11} \times 7^5 \times 11^2 = 2^{22} \times 7^5 \times 11^2$
Total number of prime factors = (22 + 5 + 2) = 29.

Ex.12. Simplify : (i) 896 x 896 - 204 x 204
(ii) 387 x 387 + 114 x 114 + 2 x 387 x 114
(iii) 81 X 81 + 68 X 68-2 x 81 X 68.

Sol.

(i) Given exp = $(896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.

(ii) Given exp = $(387)^2 + (114)^2 + (2 \times 387 \times 114)$
$= a^2 + b^2 + 2ab$, where a = 387,b=114
$= (a+b)^2 = (387 + 114)^2 = (501)^2 = 251001$.

(iii) Given exp = $(81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, Where a =81,b=68
$= (a-b)^2 = (81 - 68)^2 = (13)^{2 = 169.}$

Ex.13. Which of the following numbers is divisible by 3 ?
(i) 541326 (ii) 5967013

Sol.

(i) Sum of digits in 541326 = (5 + 4 + 1 + 3 + 2 + 6) = 21, which is divisible by 3.
Hence, 541326 is divisible by 3.

(ii) Sum of digits in 5967013 =(5+9 + 6 + 7 + 0+1 +3) = 31, which is not divisible by 3.
Hence, 5967013 is not divisible by 3.

Ex.14.What least value must be assigned to * so that the number 197*5462 is r 9 ?

Sol.

Let the missing digit be x.
Sum of digits = (1 + 9 + 7 + x + 5 + 4 + 6 +»2) = (34 + x).
For (34 + x) to be divisible by 9, x must be replaced by 2 .
Hence, the digit in place of * must be 2.

Ex. 15. Which of the following numbers is divisible by 4 ?
(i) 67920594 (ii) 618703572

Sol.

(i) The number formed by the last two digits in the given number is 94, which is not divisible by 4.
Hence, 67920594 is not divisible by 4.

(ii) The number formed by the last two digits in the given number is 72, which is divisible by 4.
Hence, 618703572 is divisible by 4.

Ex. 16. Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5 ?

Sol.

Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and $ are 4 and 0 respectively.

Ex. 17. Show that 4832718 is divisible by 11.

Sol. (Sum of digits at odd places) - (Sum of digits at even places)

$= (8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

Hence, 4832718 is divisible by 11.

Ex. 18. Is 52563744 divisible by 24 ?

Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.

Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes. So, it is divisible by 3 x 8, i.e., 24.

Ex. 19. What least number must be added to 3000 to obtain a number exactly divisible by 19 ?

Sol. On dividing 3000 by 19, we get 17 as remainder.

$\therefore$ Number to be added = (19 - 17) = 2.

Ex. 20. What least number must be subtracted from 2000 to get a number exactly divisible by 17 ?

Sol. On dividing 2000 by 17, we get 11 as remainder.

$\therefore$ Required number to be subtracted = 11.

Ex. 21. Find the number which is nearest to 3105 and is exactly divisible by 21.

Sol. On dividing 3105 by 21, we get 18 as remainder.

$\therefore$ Number to be added to 3105 = (21 - 18) - 3.

Hence, required number = 3105 + 3 = 3108.

Ex. 22. Find the smallest number of 6 digits which is exactly divisible by 111.

Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

$\therefore$ Number to be added = (111 - 100) - 11.

Hence, required number = 100011.-

Ex. 23. On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.

Sol. $$\text{Divisor} = \frac{\text{Dividend - Remainder}}{\text{.Quotient}} = \frac{15968-37}{89} = 179.$$

Ex. 24. A number when divided by 342 gives a remainder 47. When the same number ift divided by 19, what would be the remainder ?

Sol. On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then, number – 342k + 47 = (19 x 18k + 19 x 2 + 9) = 19 (18k + 2) + 9.

$\therefore$ The given number when divided by 19, gives (18k + 2) as quotient and 9 as remainder.

Ex. 25. A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed,

Sol.

3	X	
5	y	- 1
8	z	- 4
	1	- 7

$\therefore$ z = (8 x 1 + 7) = 15; y = {5z + 4) = (5 x 15 + 4) = 79; x = (3y + 1) = (3 x 79 + 1) = 238.

Now,

8	238	
5	29	- 6
3	5	- 4
	1	- 9,

$\therefore$ Respective remainders are 6, 4, 2.

Ex. 26. Find the remainder when 2^{31} is divided by 5.

Sol. 2^{10} = 1024. Unit digit of 2^{10} x 2^{10} x 2^{10} is 4 [as 4 x 4 x 4 gives unit digit 4].

$\therefore$ Unit digit of 231 is 8.

Now, 8 when divided by 5, gives 3 as remainder.

Hence, 231 when divided by 5, gives 3 as remainder.

Ex. 27. How many numbers between 11 and 90 are divisible by 7 ?

Sol. The required numbers are 14, 21, 28, 35, ... 77, 84.

This is an A.P. with a = 14 and d = (21 - 14) = 7.

Let it contain n terms.

Then, $T_n = 84$ => a + (n - 1) d = 84

=> 14 + (n - 1) x 7 = 84 or n = 11.

$\therefore$ Required number of terms = 11.

Ex. 28. Find the sum of all odd numbers upto 100.

Sol. The given numbers are 1, 3, 5, 7, ..., 99.

This is an A.P. with a = 1 and d = 2.

Let it contain n terms. Then,

1 + (n - 1) x 2 = 99 or n = 50.

$\therefore$ Required sum = $\frac{n}{2}$ (first term + last term)

= $\frac{50}{2}(1 + 99) = 2500$.

Ex. 29. Find the sum of all 2 digit numbers divisible by 3.

Sol. All 2 digit numbers divisible by 3 are :

12, 51, 18, 21, ..., 99.

This is an A.P. with a = 12 and d = 3.

Let it contain n terms. Then,

12 + (n - 1) x 3 = 99 or n = 30.

$\therefore$ Required sum = $\frac{30}{2} \times (12+99) = 1665$.

Ex.30.How many terms are there in 2,4,8,16……1024?

Sol. Clearly 2,4,8,16……..1024 form a GP. With a=2 and r = 4/2 =2.

Let the number of terms be n . Then

$2 \times 2^{n-1} = 1024$ or $2^{n-1} = 512 = 2^9$.

$\therefore$ n-1=9 or n=10.

Ex. 31. $2 + 2^2 + 2^3 + ... + 2^8 = ?$

Sol. Given series is a G.P. with a = 2, r = 2 and n = 8.

$\therefore$ sum = $\frac{a(r^n-1)}{(r-1)} = \frac{2 \times (2^8 - 1)}{(2-1)} = (2 \times 255) = 510$

2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

I. Factors and Multiples : If a number a divides another number b exactly, we say that a is a factor of b. In this case, b is called a multiple of a.

II. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.): The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are *two methods* of finding the H.C.F. of a given set of numbers :

1. Factorization Method : Express each one of the given numbers as the product of prime factors.The product of least powers of common prime factors gives H.C.F.

2. Division Method: Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

III. Least Common Multiple (L.C.M.) : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. Factorization Method of Finding L.C.M.: Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors,

2. Common Division Method {Short-cut Method) of Finding L.C.M.: Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers,

IV. Product of two numbers =Product of their H.C.F. and L.C.M.

V. Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.

VI. H.C.F. and L.C.M. of Fractions:

1. $\text{H C F} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$ 2. $\text{L C M} = \dfrac{\text{L.C.M of Numerators}}{\text{H.C.F. of Denominators}}$

VII. H.C.F. and L.C.M. of Decimal Fractions: In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

VIII. Comparison of Fractions: Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

SOLVED EXAMPLES

Ex. 1. Find the H.C.F. of 2^3 X 3^2 X 5 X 7^4, 2^2 X 3^5 X 5^2 X 7^3,2^3 X 5^3 X 7^2

Sol. The prime numbers common to given numbers are 2,5 and 7.

H.C.F. = 2^2 x 5 x7^2 = 980.

Ex. 2. Find the H.C.F. of 108, 288 and 360.

Sol. 108 = 2^2 x 3^3, 288 = 2^5 x 3^2 and 360 = 2^3 x 5 x 3^2.

H.C.F. = 2^2 x 3^2 = 36.

Ex. 3. Find the H.C.F. of 513, 1134 and 1215.

Sol.

```
1134 ) 1215 ( 1
        1134
        ----------
           81 ) 1134 ( 14
                 81
                 324
                 324
                 ---
                  x
```

$\therefore$H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

```
81 )  513 ( 6
       486
      -------
        27) 81 ( 3
             81
            ----
              0
```

H.C.F. of given numbers = 27.

Ex. 4. Reduce $\frac{391}{667}$ to lowest terms.

to lowest terms.

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}$

Ex.5.Find the L.C.M. of 2^2 x 3^3 x 5 x 7^2, 2^3 x 3^2 x 5^2 x 7^4 , 2 x 3 x $5^{3 \text{ x } 7 \text{ x } 11.}$

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 = 2^3 x 3^3 x 5^3 x 7^4 x 11

Ex.6. Find the L.C.M. of 72, 108 and 2100.

Sol. 72 = 2^3 x 3^2, 108 = 3^3 x 2^2, 2100 = 2^2 x 5^2 x 3 x 7.

L.C.M. = 2^3 x 3^3 x 5^2 x 7 = 37800.

Ex.7.Find the L.C.M. of 16, 24, 36 and 54.

Sol.

2	16 - 24 - 36 - 54
2	8 - 12 - 18 - 27
2	4 - 6 - 9 - 27
3	2 - 3 - 9 - 27
3	2 - 1 - 3 - 9
	2 - 1 - 1 - 3

$\therefore$ L.C.M. = 2 x 2 x 2 x 3 x 3 x 2 x 3 = 432.

Ex. 8. Find the H.C.F. and L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

Sol. H.C.F. of given fractions = $\frac{\text{H.C.F. of } 2,8,16,10}{\text{L.C.M. of } 3,9,81,27} = \frac{2}{81}$

L.C.M of given fractions = $\frac{\text{L.C.M. of } 2,8,16,10}{\text{H.C.F. of } 3,9,81,27} = \frac{80}{3}$

Ex. 9. **Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.**

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, find the numbers.

Sol. Let the required numbers be 15.x and llx.

Then, their H.C.F. is x. So, x = 13.

The numbers are (15 x 13 and 11 x 13) i.e., 195 and 143.

Ex. 11. TheH.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77,find the other.

Sol. Other number = $\frac{11 \text{ X } 693}{77} = 99$

Ex. 12. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$, $1665 = 3^2 \times 5 \times 37$.

$\therefore$ H.C.F. = 32 x 5 = 45.

Hence, required length = 45 cm.

Ex. 13. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

Sol. Required number = H.C.F. of (1657 - 6) and (2037 - 5) = H.C.F. of 1651 and 2032

```
1651 )  2032 ( 1 1651
         1651
          381 )  1651 ( 4
                 1524
                  127 )  381 ( 3
                         381
                           0
```

Required number = 127.

Ex. 14. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

Sol . Required number = H.C.F. of (132 - 62), (237 - 132) and (237 - 62)

= H.C.F. of 70, 105 and 175 = 35.

Ex.15.Find the least number exactly divisible by 12,15,20,27.

Sol.

3	12	-	15	-	20	-	27
4	4	-	5	-	20	-	9
5	1	-	5	-	5	-	9
	1	-	1	-	1	-	9

Ex.16.Find the least number which when divided by 6,7,8,9, and 12 leave the same remainder 1 each case
Sol. Required number = (L.C.M OF 6,7,8,9,12) + 1

3	6	- 7	- 8	- 9	- 12
4	2	- 7	- 8	- 3	- 4
5	1	- 7	- 4	- 3	- 2
	1	- 7	- 2	- 3	- 1

$\therefore$ L.C.M = 3 X 2 X 2 X 7 X 2 X 3 = 504.
Hence required number = (504 +1) = 505.

Ex.17. Find the largest number of four digits exactly divisible by 12,15,18 and 27.
Sol. The Largest number of four digits is 9999.
Required number must be divisible by L.C.M. of 12,15,18,27 i.e. 540.
On dividing 9999 by 540,we get 279 as remainder .
$\therefore$ Required number = (9999-279) = 9720.

Ex.18.Find the smallest number of five digits exactly divisible by 16,24,36 and 54.
Sol. Smallest number of five digits is 10000.
Required number must be divisible by L.C.M. of 16,24,36,54 i.e 432,
On dividing 10000 by 432,we get 64 as remainder.
$\therefore$ Required number = 10000 +(432 – 64) = 10368.

Ex.19.Find the least number which when divided by 20,25,35 and 40 leaves remainders 14,19,29 and 34 respectively.
Sol. Here,(20-14) = 6,(25 – 19)=6,(35-29)=6 and (40-34)=6.
$\therefore$ Required number = (L.C.M. of 20,25,35,40) – 6 =1394.

Ex.20.Find the least number which when divided by 5,6,7, and 8 leaves a remainder 3, but when divided by 9 leaves no remainder .
Sol. L.C.M. of 5,6,7,8 = 840.
$\therefore$ Required number is of the form 840k + 3
Least value of k for which (840k + 3) is divisible by 9 is k = 2.
$\therefore$ Required number = (840 X 2 + 3)=1683

Ex.21.The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 sec.respectively .If they all change simultaneously at 8:20:00 hours,then at what time they again change simultaneously .
Sol. Interval of change = (L.C.M of 48,72,108)sec.=432sec.
So, the lights will agin change simultaneously after every 432 seconds i.e,7 min.12sec
Hence , next simultaneous change will take place at 8:27:12 hrs.

Ex.22.Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in the ascending order.

Sol.L.C.M. of 18,36,45 and 60 = 180.

Now, $\frac{17}{18} = \frac{17 \times 10}{18 \times 10} = \frac{170}{180}$; $\frac{31}{36} = \frac{31 \times 5}{36 \times 5} = \frac{155}{180}$;

$\frac{43}{45} = \frac{43 \times 4}{45 \times 4} = \frac{172}{180}$; $\frac{59}{60} = \frac{59 \times 3}{60 \times 3} = \frac{177}{180}$;

Since, 155<170<172<177, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$

Hence, $\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$

3. DECIMAL FRACTIONS

IMPORTANT FACTS AND FORMULAE

I. Decimal Fractions : Fractions in which denominators are powers of 10 are known as **decimal fractions.**

Thus ,1/10=1 tenth=.1;1/100=1 hundredth =.01;

99/100=99 hundreths=.99;7/1000=7 thousandths=.007,etc

II. Conversion of a Decimal Into Vulgar Fraction : Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.

Thus, 0.25=25/100=1/4;2.008=2008/1000=251/125.

III. 1. Annexing zeros to the extreme right of a decimal fraction does not change its value
Thus, 0.8 = 0.80 = 0.800, etc.

2. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.
Thus, 1.84/2.99 = 184/299 = 8/13; 0.365/0.584 = 365/584=5

IV. Operations on Decimal Fractions :

1. Addition and Subtraction of Decimal Fractions : The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

2. **Multiplication of a Decimal Fraction By a Power of 10** : Shift the decimal point to the right by as many places as is the power of 10.
Thus, 5.9632 x 100 = 596,32; 0.073 x 10000 = 0.0730 x 10000 = 730.

3.**Multiplication of Decimal Fractions** : Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.
Suppose we have to find the product (.2 x .02 x .002). Now, 2x2x2 = 8. Sum of decimal places = (1 + 2 + 3) = 6. .2 x .02 x .002 = .000008.

4.**Dividing a Decimal Fraction By a Counting Number :** Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.
Suppose we have to find the quotient (0.0204 + 17). Now, 204 ^ 17 = 12. Dividend contains 4 places of decimal. So, 0.0204 + 17 = 0.0012.

5. **Dividing a Decimal Fraction By a Decimal Fraction :** Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, 0.00066/0.11 = (0.00066*100)/(0.11*100) = (0.066/11) = 0.006V

V. Comparison of Fractions : Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions 3/5, 6/7 and 7/9 in descending order.

now, 3/5=0.6,6/7 = 0.857,7/9 = 0.777....

since 0.857>0.777...>0.6, so 6/7>7/9>3/5

VI. Recurring Decimal : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a *recurring decimal.*

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set

Thus 1/3 = 0.3333….= 0.3; 22 /7 = 3.142857142857.....= $3.\overline{142857}$

Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

thus ,0.5 = 5/9; 0.53 = 53/59 ;0.067 = 67/999;etc...

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., 0.17333.= 0.173.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated, In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus 0.16 = (16-1) / 90 = 15/19 = 1/6;

$0.\overline{2273}$ = (2273 – 22)/9900 = 2251/9900

VII. Some Basic Formulae :

1. $(a + b)(a - b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$.
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$.
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$.
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

SOLVED EXAMPLES

Ex. 1. Convert the following into vulgar fraction:
(i) 0.75 (ii) 3.004 (iii) 0.0056

Sol. (i). 0.75 = 75/100 = 3/4 (ii) 3.004 = 3004/1000 = 751/250 (iii) 0.0056 = 56/10000 = 7/1250

Ex. 2. Arrange the fractions 5/8, 7/12, 13/16, 16/29 and 3/4 in ascending order of magnitude.

Sol. Converting each of the given fractions into decimal form, we get :
5/8 = 0.624, 7/12 = 0.8125, 16/29 = 0.5517, and 3/4 = 0.75
Now, 0.5517<0.5833<0.625<0.75<0.8125
$\therefore$ 16/29 < 7/12 < 5/8 < 3/4 < 13/16

Ex. 3. arrange the fractions 3/5, 4/7, 8/9, and 9/11 in their descending order.

Sol. Clearly, 3/5 = 0.6, 4/7 = 0.571, 8/9 = 0.88, 9/111 = 0.818.
Now, 0.88 > 0.818 > 0.6 > 0.571
$\therefore$ 8/9 > 9/11 > 3/4 > 13/ 16

Ex. 4. Evaluate : (i) 6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025
(ii) 5.064 + 3.98 + 0.7036 + 7.6 + 0.3 + 2

Sol. (i)
$$\begin{array}{r} 6202.5 \\ 620.25 \\ 62.025 \\ 6.2025 \\ +\ \underline{0.62025} \\ \underline{6891.59775} \end{array}$$

(ii)
$$\begin{array}{l} 5.064 \\ 3.98 \\ 0.7036 \\ 7.6 \\ 0.3 \\ \underline{2.0} \\ \underline{19.6476} \end{array}$$

Ex. 5. Evaluate : (i) 31.004 – 17.2368 (ii) 13 – 5.1967

Sol. (i) 31.0040 (ii) 31.0000

$$\begin{array}{r} 31.0040 \\ -\underline{17.2386} \\ \underline{13.7654} \end{array} \qquad \begin{array}{r} 31.0000 \\ -\underline{\ 5.1967} \\ \underline{7.8033} \end{array}$$

Ex. 6. What value will replace the question mark in the following equations ?

(i) 5172.49 + 378.352 + ? = 9318.678

(ii) ? – 7328.96 + 5169.38

Sol. (i) Let 5172.49 + 378.352 + x = 9318.678

Then , x = 9318.678 – (5172.49 + 378.352) = 9318.678 – 5550.842 = 3767.836

(ii) Let x – 7328.96 = 5169.38. Then, x = 5169.38 + 7328.96 = 12498.34.

Ex. 7. Find the products: (i) 6.3204 * 100 (ii) 0.069 * 10000

Sol. (i) 6.3204 * 1000 = 632.04 (ii) 0.069 * 10000 = 0.0690 * 10000 = 690

Ex. 8. Find the product:

(i) 2.61 * 1.3 (ii) 2.1693 * 1.4 (iii) 0.4 * 0.04 * 0.004 * 40

Sol. (i) 261 8 13 = 3393. Sum of decimal places of given numbers = (2+1) = 3.

2.61 * 1.3 = 3.393.

(ii) 21693 * 14 = 303702. Sum of decimal places = (4+1) = 5

2.1693 * 1.4 = 3.03702.

(iii) 4 * 4 * 4 * 40 = 2560. Sum of decimal places = (1 + 2+ 3) = 6

0.4 * 0.04 * 0.004 * 40 = 0.002560.

Ex. 9. Given that 268 * 74 = 19832, find the values of 2.68 * 0.74.

Sol. Sum of decimal places = (2 + 2) = 4

2.68 * 0.74 = 1.9832.

Ex. 10. Find the quotient:

(i) 0.63 / 9 (ii) 0.0204 / 17 (iii) 3.1603 / 13

Sol. (i) 63 / 9 = 7. Dividend contains 2 places decimal.

0.63 / 9 = 0.7.

(ii) 204 / 17 = 12. Dividend contains 4 places of decimal.

0.2040 / 17 = 0.0012.

(iii) 31603 / 13 = 2431. Dividend contains 4 places of decimal.

3.1603 / 13 = 0.2431.

Ex. 11. Evaluate :

(i) 35 + 0.07 (ii) 2.5 + 0.0005

(iii) 136.09 + 43.9

Sol. (i) 35/0.07 = (35*100) / (0.07*100) = (3500 / 7) = 500
(ii) 25/0.0005 = (25*10000) / (0.0005*10000) = 25000 / 5 = 5000
(iii) 136.09/43.9 = (136.09*10) / (43.9*10) = 1360.9 / 439 = 3.1

Ex. 12. What value will come in place of question mark in the following equation?
(i) 0.006 +? = 0.6 **(ii) ? + 0.025 = 80**

Sol. (i) Let 0.006 / x = 0.6, Then, x = (0.006 / 0.6) = (0.006*10) / (0.6*10) = 0.06/6 = 0.01
(ii) Let x / 0.025 = 80, Then, x = 80 * 0.025 = 2

Ex. 13. If (1 / 3.718) = 0.2689, Then find the value of (1 / 0.0003718).

Sol. (1 / 0.0003718) = (10000 / 3.718) = 10000 * (1 / 3.718) = 10000 * 0.2689 = 2689.

Ex. 14. Express as vulgar fractions : (i) $0.\overline{37}$ (ii) $0.\overline{053}$ (iii) $3.\overline{142857}$

Sol. (i) $0.\overline{37}$ = 37 / 99 . (ii) $0.\overline{053}$ = 53 / 999

(iii) $3.\overline{142857}$ = 3 + $0.\overline{142857}$ = 3 + (142857 / 999999) = 3 (142857/999999)

Ex. 15. Express as vulgar fractions : (i) $0.1\overline{7}$ (ii) $0.12\overline{54}$ (iii) $2.53\overline{6}$

Sol. (i) $0.1\overline{7}$ = (17 – 1)/90 = 16 / 90 = 8/ 45

(ii) $0.12\overline{54}$ = (1254 – 12)/ 9900 = 1242 / 9900 = 69 / 550

(iii) 2.536 = 2 + 0.536 = 2 + (536 – 53)/900 = 2 + (483/900) = 2 + (161/300) = 2 (161/300)

Ex. 16. Simplify:
$$\frac{\mathbf{0.05 * 0.05 * 0.05 + 0.04 * 0.04 * 0.04}}{\mathbf{0.05 * 0.05 - 0.05 * 0.04 + 0.04 * 0.04}}$$

Sol. Given expression = $(a^3 + b^3) / (a^2 - ab + b^2)$, where a = 0.05 , b = 0.04
= (a +b) = (0.05 +0.04) =0.09

4. SIMPLIFICATION

IMPORTANT CONCEPTS

I. ’BODMAS’Rule: This rule depicts the correct sequence in which the operations are to be executed,so as to find out the value of a given expression.

Here, ‘B’ stands for ’bracket’ ,’O’for ‘of’ , ‘D’ for’ division’ and ‘M’ for ‘multiplication’, ‘A’ for ‘addition’ and ‘S’ for ‘subtraction’.
Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order(), {} and [].

After removing the brackets, we must use the following operations strictly in the order:
(1)of (2)division (3) multiplication (4)addition (5)subtraction.

II. Modulus of a real number : Modulus of a real number a is defined as

$|a| = \{a$, if $a>0$
$\quad -a$, if $a<0$

Thus, $|5|=5$ and $|-5|=-(-5)=5$.
III. Virnaculum (or bar): When an expression contains Virnaculum, before applying the ‘BODMAS’ rule, we simplify the expression under the Virnaculum.

SOLVED EXAMPLES

Ex. 1. Simplify: (i)5005-5000+10 (ii) 18800+470+20

Sol. (i)5005-5000+10=5005-(5000/10)=5005-500=4505.

(ii)18800+470+20=(18800/470)+20=40/20=2.

Ex. 2. Simplify: b-[b-(a+b)-{b-(b-a-b)}+2a]

Sol. Given expression=b-[b-(a+b)-{b-(b-a+b)}+2a]
=b-[b-a-b-{b-2b+a}+2a]
=b-[-a-{b-2b+a+2a}]
=b-[-a-{-b+3a}]=b-[-a+b-3a]
=b-[-4a+b]=b+4a-b=4a.

Ex. 3. What value will replace the question mark in the following equation?

$$4\frac{1}{2}+3\frac{1}{6}+?+2\frac{1}{3}=13\frac{2}{5}.$$

Sol. Let 9/2+19/6+x+7/3=67/5

Then x=(67/5)-(9/2+19/6+7/3)⇔x=(67/5)-((27+19+14)/6)=((67/5)-(60/6)
⇔x=((67/5)-10)=17/5=$3\frac{2}{5}$

Hence, missing fractions =$3\frac{2}{5}$

Ex.4. 4/15 of 5/7 of a number is greater than 4/9 of 2/5 of the same number by 8. What is half of that number?

Sol. Let the number be x. then 4/15 of 5/7 of x-4/9 of 2/5 of x=8⇔4/21x-8/45x=8
⇔(4/21-8/45)x=8⇔(60-56)/315x=8⇔4/315x=8
⇔x=(8*315)/4=630⇔1/2x=315
Hence required number = 315.

Ex. 5. Simplify: $\left[3\frac{1}{4}\div\{1\frac{1}{4}-1/2(2\frac{1}{2}-1/4-\overline{1/6})\}\right]$

Sol. Given exp. =[13/4÷{5/4-1/2(5/2-(3-2)/12)}]=[13/4÷{5/4-1/2(5/2-1/12)}]
=[13/4÷{5/4-1/2((30-1)/12)}]=[13/4÷{5/4-29/24}]
=[13/4÷{(30-39)/24}]=[13/4÷1/24]=[(13/4)*24]=78

Ex. 6. Simplify: 108÷36of $\frac{1}{4}$+$\frac{2}{5}$*3 $\frac{1}{4}$

Sol. Given exp.= 108÷9+$\frac{2}{5}$*$\frac{13}{4}$ =$\frac{108}{9}$+$\frac{13}{10}$ =12+$\frac{13}{10}$ $\left(=\frac{133}{10}=\right)$ 13 $\frac{3}{10}$

Ex.7 Simplify: $\frac{(7/2)\div(5/2)*(3/2)}{(7/2)\div(5/2)\text{of }(3/2)}$ **÷ 5.25**

sol.
Given exp. $\frac{(7/2)\times(2/5)\times(3/2)}{(7/2)\div(15/4)}$ ÷ 5.25=(21/10)÷(525/100)=(21/10)×(15/14)

Ex. 8. Simplify: (i) 12.05*5.4+0.6 (ii) 0.6*0.6+0.6*0.6 (Bank P.O 2003)

Sol. (i) Given exp. = 12.05*(5.4/0.6) = (12.05*9) = 108.45
(ii) Given exp. = 0.6*0.6+(0.6*6) = 0.36+0.1 = 0.46

Ex. 9. Find the value of x in each of the following equation:
(i) [(17.28/x) / (3.6*0.2)] = 2
(ii) 3648.24 + 364.824 + x – 36.4824 = 3794.1696
(iii) 8.5 – { 5 ½ – [7 ½ + 2.8]/x}*4.25/$(0.2)^2$ = 306 (Hotel Management,1997)

Sol. (i) (17.28/x) = 2*3.6*0.2 ⇔ x = (17.28/1.44) = (1728/14) = 12.
(ii) (364.824/x) = (3794.1696 + 36.4824) – 3648.24 = 3830.652 – 3648.24 = 182.412.

$\Leftrightarrow$ x = (364.824/182.412) =2.

(iii) 8.5-{5.5-(7.5+(2.8/x))}*(4.25/0.04) = 306
$\Leftrightarrow$ 8.5-{5.5-{(7.5x+2.8)/x)}*(425/4) = 306
$\Leftrightarrow$ 8.5-{(5.5x-7.5x-2.8)/x}*(425/4) = 306
$\Leftrightarrow$ 8.5-{(-2x-2.8)/x}*106.25 = 306
$\Leftrightarrow$ 8.5-{(-212.5x-297.5)/x} = 306
$\Leftrightarrow$ (306-221)x = 297.5 $\Leftrightarrow$ x =(297.5/85) = 3.5.

Ex. 10. If (x/y)=(6/5), find the value $(x^2+y^2)/(x^2-y^2)$

Sol. $(x^2+y^2)/(x^2-y^2) = (x^2/y^2+1)/(x^2/y^2-1) = [(6/5)^2+1] / [(6/5)^2-1]$
= [(36/25)+1] / [(36/25)-1] = (61*25)/(25*11) = 61/11

Ex. 11. Find the value of $\mathbf{4 - \dfrac{5}{1+\dfrac{1}{3+\dfrac{1}{2+\dfrac{1}{4}}}}}$

Sol. Given exp. = $4 - \dfrac{5}{1+\dfrac{1}{3+\dfrac{1}{2+\dfrac{1}{4}}}} = 4 - \dfrac{5}{1+\dfrac{1}{3+\dfrac{4}{9}}} = 4 - \dfrac{5}{1+\dfrac{1}{(31/9)}}$

$= 4 - \dfrac{5}{1+\dfrac{9}{31}} = 4 - \dfrac{5}{(40/31)} = 4 - (5*31)/40 = 4 - (31/8) = 1/8$

Ex. 12. If $\mathbf{\dfrac{2x}{1+\dfrac{1}{1+\dfrac{x}{1-x}}}}$ **= 1 ., then find the value of x .**

Sol. We have : $\dfrac{2x}{1+\dfrac{1}{\dfrac{(1-x)-x}{1-x}}} = 1 \Leftrightarrow \dfrac{2x}{1+\dfrac{1}{[1/(1-x)]}} = 1 \Leftrightarrow \dfrac{2x}{1+(1-x)} = 1$

$\Leftrightarrow$ 2x = 2-x $\Leftrightarrow$ 3x = 2 $\Leftrightarrow$ x = (2/3).

Ex.13.(i)If a/b=3/4 and 8a+5b=22,then find the value of a.
(ii)if x/4-x-3/6=1,then find the value of x.

Sol. (i) (a/b)=3/4 $\Rightarrow$b=(4/3) a.
$\therefore$ 8a+5b=22 $\Rightarrow$ 8a+5*(4/3) a=22 $\Rightarrow$ 8a+(20/3) a=22
$\Rightarrow$44a = 66 $\Rightarrow$ a=(66/44)=3/2
(ii) (x /4)-((x-3)/6)=1$\Leftrightarrow$ (3x-2(x-3))/12 = 1 $\Leftrightarrow$ 3x-2x+6=12 $\Leftrightarrow$ x=6.

Ex.14.If 2x+3y=34 and ((x + y)/y)=13/8,then find the value of 5y+7x.

Sol. The given equations are:
2x+3y=34 …(i) and, ((x + y) /y)=13/8 $\Rightarrow$ 8x+8y=13y $\Rightarrow$ 8x-5y=0 …(ii)
Multiplying (i) by 5,(ii) by 3 and adding, we get : 34x=170 or x=5.
Putting x=5 in (i), we get: y=8.
$\therefore$ 5y+7x=((5*8)+(7*5))=40+35=75

Ex.15.If 2x+3y+z=55,x-y=4 and y - x + z=12,then what are the values of x , y and z?

Sol. The given equations are:
2x+3y+z=55 …(i); x + z - y=4 …(ii); y -x + z =12 …(iii)
Subtracting (ii) from (i), we get: x+4y=51 …(iv)
Subtracting (iii) from (i), we get: 3x+2y=43 …(v)
Multiplying (v) by 2 and subtracting (iv) from it, we get: 5x=35 or x=7.
Putting x=7 in (iv), we get: 4y=44 or y=11.
Putting x=7,y=11 in (i), we get: z=8.

Ex.16.Find the value of (1-(1/3))(1-(1/4))(1-(1/5))….(1-(1/100)).

Sol. Given expression = (2/3)*(3/4)*(4/5) *…….* (99/100) = 2/100 = 1/50.

Ex.17. Find the value of (1/(2*3))+(1/(3*4))+(1/(4*5))+(1/(5*6))+…..+ ((1/(9*10)).

Sol. Given expression=((1/2)-(1/3))+((1/3)-(1/4))+((1/4)-(1/5))+
((1/5)-(1/6))+….+ ((1/9)-(1/10))
=((1/2)-(1/10))=4/10 = 2/5.

Ex.18.Simplify: $99^{48}/_{49}$ * 245.

Sol. Given expression = (100-1/49) * 245=(4899/49) * 245 = 4899 * 5=24495.

Ex.19.A board 7ft. 9 inches long is divided into 3 equal parts . What is the length of each part?

Sol. Length of board=7ft. 9 inches=(7*12+9)inches=93 inches.
$\therefore$ Length of each part = (93/3) inches = 31 inches = 2ft. 7 inches

20.A man divides Rs. Among 5 sons,4daughters and 2 nephews .If each daughter receives four times as much as each nephews and each son receives five times as much as each nephews ,how much does each daughter receive?
Let the share of each nephews be Rs.x.
Then,share of each daughter=rs4x;share of each son=Rs.5x;
So,5*5x+4*4x+2*x=8600

25x+16x+2x=8600
=43x=8600
x=200;

21.A man spends 2/5 of his salary on house rent,3/10 of his salary on food and 1/8 of his salary on conveyence.if he has Rs.1400 left with him,find his expenditure on food and conveyence.
Part of salary left=1-(2/5+3/10+1/8)
Let the monthly salary be Rs.x
Then, 7/40 of x=1400
X=(1400*40/7)
=8600
Expenditure on food=Rs.(3/10*800)=Rs.2400
Expenditure on conveyence=Rs.(1/8*8000)=Rs.1000

22.A third of Arun's marks in mathematics exeeds a half of his marks in english by 80.if he got 240 marks In two subjects together how many marks did he got inh english?
Let Arun's marks in mathematics and english be x and y
Then 1/3x-1/2y=30
2x-3y=180……>(1)
x+y=240…….>(2)
solving (1) and (2)
x=180
and y=60

23.A tin of oil was 4/5full.when 6 bottles of oil were taken out and four bottles of oil were poured into it, it was ¾ full. how many bottles of oil can the tin contain?
Suppose x bottles can fill the tin completely
Then4/5x-3/4x=6-4
X/20=2
X=40
Therefore required no of bottles =40

24.if 1/8 of a pencil is black ½ of the remaining is white and the remaining 3 ½ is blue find the total length of the pencil?
Let the total length be xm
Then black part =x/8cm
The remaining part=(x-x/8)cm=7x/8cm
White part=(1/2 *7x/8)=7x/16 cm
Remaining part=(7x/8-7x/16)=7x/16cm
7x/16=7/2
x=8cm

25.in a certain office 1/3 of the workers are women ½ of the women are married and 1/3 of the married women have children if ¾ of the men are married and 2/3 of the married men have children what part of workers are without children?

Let the total no of workers be x
No of women =x/3
No of men =x-(x/3)=2x/3
No of women having children =1/3 of ½ ofx/3=x/18
No of men having children=2/3 of ¾ of2x/3=x/3
No of workers having children = x/8 +x/3=7x/18
Workers having no children=x-7x/18=11x/18=11/18 of all wprkers

26.a crate of mangoes contains one bruised mango for every thirty mango in the crate. If three out of every four bruised mango are considerably unsaleble and there are 12 unsaleable mangoes in the crate then how msny mango are there in the crate?

Let the total no of mangoes in the crate be x
Then the no of bruised mango = 1/30 x
Let the no of unsalable mangoes =3/4 (1/30 x)
1/40 x =12
x=480

27. a train starts full of passengers at the first station it drops 1/3 of the passengers and takes 280more at the second station it drops one half the new total and takes twelve more .on arriving at the third station it is found to have 248 passengers. Find the no of passengers in the beginning?
Let no of passengers in the beginning be x
After first station no passengers=(x-x/3)+280=2x/3 +280
After second station no passengers =1/2(2x/3+280)+12
½(2x/3+280)+12=248
2x/3+280=2*236
2x/3=192
x=288

28.if a^2+b^2=177and ab=54 then find the value of a+b/a-b?
$(a+b)^2=a^2+b^2+2ab$=117+2*24=225
a+b=15
$(a-b)^2=a^2+b^2-2ab$=117-2*54
a-b=3
a+b/a-b=15/3=5

29.find the value of (75983*75983- 45983*45983/30000)
Given expression=$(75983)^2-(45983)^2$/(75983-45983)
=$(a-b)^2$/(a-b)
=(a+b)(a-b)/(a-b)
=(a+b)
=75983+45983
=121966

30.find the value of $\left[\dfrac{\mathbf{343*343*343-113*113*113}}{\mathbf{343*343+343*113+113*113}}\right]$

Given expression= $\dfrac{(a^3-b^3)}{a^2+ab+b^2}$

=(a-b)

=(343-113)

.=230

31.Village X has a population of 68000,which is decreasing at the rate of 1200 per year.VillagyY has a population of 42000,which is increasing at the rate of 800 per year .in how many years will the population of the two villages be equal?

Let the population of two villages be equal after p years

Then,68000-1200p=42000+800p

2000p=26000

p=13

32.From a group of boys and girls,15 girls leave.There are then left 2 boys for each girl.After this,45 boys leave.There are then 5 girls for each boy.Find the number of girls in the beginning?

Let at present there be x boys.

Then,no of girls at present=5x

Before the boys had left:no of boys=x+45

And no of girls=5x

X+45=2*5x

9x=45

x=5

no of girls in the beginning=25+15=40

33.An employer pays Rs.20 for each day a worker works and for feits Rs.3 for each day is ideal at the end of sixty days a worker gets Rs.280 . for how many days did the worker remain ideal?

Suppose a worker remained ideal for x days then he worked for 60-x days

20*(60-x)-3x=280

1200-23x=280

23x=920

x=40

Ex 34.kiran had 85 currency notes in all , some of which were of Rs.100 denaomination and the remaining of Rs.50 denomination the total amount of all these currency note was Rs.5000.how much amount did she have in the denomination of Rs.50?

Let the no of fifty rupee notes be x

Then,no of 100 rupee notes =(85-x)

50x+100(85-x)=5000

x+2(85-x)=100
x=70
so,,required amount=Rs.(50*70)= Rs.3500

Ex. 35. When an amount was distributed among 14 boys, each of them got rs 80 more than the amount received by each boy when the same amount is distributed equally among 18 boys. What was the amount?

Sol. Let the total amount be Rs. X the,

$$\frac{x}{14} - \frac{x}{18} = 80 \Leftrightarrow \frac{2x}{126} = 80 \Leftrightarrow \frac{x}{63} = 63 \times 80 = 5040.$$

Hence the total amount is 5040.

Ex. 36. Mr. Bhaskar is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. for how many days is Mr. Bhaskar on tour?

Sol. Suppose Mr. Bhaskar is on tour for x days. Then,

$$\frac{360}{x} - \frac{360}{x+4} = 3 \Leftrightarrow \frac{1}{x} - \frac{1}{x+4} = \frac{1}{120} \Leftrightarrow x(x+4) = 4 \times 120 = 480$$

$$\Leftrightarrow x^2 + 4x - 480 = 0 \Leftrightarrow (x+24)(x-20) = 0 \Leftrightarrow x = 20.$$

Hence Mr. Bhaskar is on tour for 20 days.

Ex. 37. Two pens and three pencils cost Rs 86. four Pens and a pencil cost Rs. 112. find the cost of a pen and that of a pencil.

Sol. Let the cost of a pen ands a pencil be Rs. X and Rs. Y respectively.

Then, 2x + 3y = 86 ….(i) and 4x + y =112.

Solving (i) and (ii), we get: x = 25 and y = 12.

Cost of a pen =Rs. 25 and the cost of a pencil =Rs. 12.

Ex. 38. Arjun and Sajal are friends . each has some money. If Arun gives Rs. 30 to Sajal, the Sajal will have twice the money left with Arjun. But, if Sajal gives Rs. 10 to Arjun, the Arjun will have thrice as much as is left with Sajal. How much money does each have?

Sol. Suppose Arun has Rs. X and Sjal has Rs. Y. then,

2(x-30)= y+30 => 2x-y =90 …(i)

and x +10 =3(y-10) => x-3y = - 40 …(ii)

Solving (i) and (ii), we get x =62 and y =34.

Arun has Rs. 62 and Sajal has Rs. 34.

Ex. 39. In a caravan, in addition to 50 hens there are 45 goats and 8 camels with some keepers. If the total number of feet be 224 more than the number of heads, find the number of keepers.

Sol. Let the number of keepers be x then,

Total number of heads $=(50 + 45 + 8 + x)= (103 + x)$.

Total number of feet $= (45 + 8) \times 4 + (50 + x) \times 2 =(312 +2x)$.

$$(312 + 2x)-(103 + x) =224 \Leftrightarrow x =15.$$

Hence, number of keepers =15.

SQUARE ROOTS AND CUBE ROOTS

IMPORTANT FACTS AND FORMULAE

Square Root: If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.
Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.
Cube Root: The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.
Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.
Note:
1.$\sqrt{xy} = \sqrt{x} * \sqrt{y}$ 2. $\sqrt{(x/y)} = \sqrt{x} / \sqrt{y} = (\sqrt{x} / \sqrt{y}) * (\sqrt{y} / \sqrt{y}) = \sqrt{xy} / y$

SOLVED EXAMPLES

Ex. 1. Evaluate √6084 by factorization method .

Sol. **Method:** Express the given number as the product of prime factors. Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.
Thus, resolving 6084 into prime factors, we get:
$6084 = 2^2 \times 3^2 \times 13^2$
$\therefore \sqrt{6084} = (2 \times 3 \times 13) = 78.$

2	6084
2	3042
3	1521
3	507
13	169
	13

Ex. 2. Find the square root of 1471369.

Sol. **Explanation:** In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.
Now, $1^2 = 1$. On subtracting, we get 0 as remainder.
Now, bring down the next period *i.e.,* 47.
Now, trial divisor is 1 x 2 = 2 and trial dividend is 47.
So, we take 22 as divisor and put 2 as quotient.
The remainder is 3.
Next, we bring down the next period which is 13.
Now, trial divisor is 12 x 2 = 24 and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.
The remainder is 72.
Bring down the next period *i.e.,* 69.
Now, the trial divisor is 121 x 2 = 242 and the trial dividend is 7269. So, we take 3as quotient and 2423 as divisor. The remainder is then zero.
Hence, $\sqrt{1471369} = 1213$.

1	1471369 (1213
	1
22	47
	44
241	313
	241
2423	7269
	7269
	x

Ex. 3. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$.

Sol. Given expression = $\sqrt{248 + \sqrt{51 + 13}} = \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$.

Ex. 4. If a * b * c = $\sqrt{(a + 2)(b + 3)} / (c + 1)$, find the value of 6 * 15 * 3.

Sol. $6 * 15 * 3 = \sqrt{(6 + 2)(15 + 3)} / (3 + 1) = \sqrt{8 * 18} / 4 = \sqrt{144} / 4 = 12 / 4 = 3$.

Ex. 5. ***Find the value of $\sqrt{25/16}$.***

Sol. $\sqrt{25 / 16} = \sqrt{25} / \sqrt{16} = 5 / 4$

Ex. 6. ***What is the square root of 0.0009?***

Sol. $\sqrt{0.0009} = \sqrt{9 / 1000} = 3 / 100 = 0.03$.

Ex. 7. ***Evaluate $\sqrt{175.2976}$.***

Sol. Method: We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$\therefore \sqrt{175.2976} = 13.24$

1	175.2976 (13.24
	1
23	75
	69
262	629
	524
2644	10576
	10576
	x

Ex. 8. ***What will come* in *place of question mark* in *each of the following* questions?**

***(i)* $\sqrt{32.4 / ?} = 2$** ***(ii)* $\sqrt{86.49} + \sqrt{5 + (\ ?\ )^2} = 12.3$.**

Sol. (i) Let $\sqrt{32.4 / x} = 2$. Then, $32.4/x = 4 \iff 4x = 32.4 \iff x = 8.1$.

(ii) Let $\sqrt{86.49} + \sqrt{5 + x^2} = 12.3$.

Then, $9.3 + \sqrt{5+x^2} = 12.3 \iff \sqrt{5+x^2} = 12.3 - 9.3 = 3$

$\iff 5 + x^2 = 9 \iff x^2 = 9 - 5 = 4 \iff x = \sqrt{4} = 2$.

Ex.9. Find the value of $\sqrt{0.289 / 0.00121}$.

Sol. $\sqrt{0.289 / 0.00121} = \sqrt{0.28900/0.00121} = \sqrt{28900/121} = 170 / 11$.

Ex.10. If $\sqrt{1 + (x / 144)} = 13 / 12$, the find the value of x.

Sol. $\sqrt{1 + (x / 144)} = 13 / 12 \Rightarrow (1 + (x / 144)) = (13 / 12)^2 = 169 / 144$

$\Rightarrow x / 144 = (169 / 144) - 1$

$\Rightarrow x / 144 = 25/144 \Rightarrow x = 25$.

Ex. 11. Find the value of $\sqrt{3}$ up to three places of decimal.

Sol.

$$\begin{array}{r|l}
1 & 3.000000 \quad (1.732 \\
 & 1 \\ \hline
27 & 200 \\
 & 189 \\ \hline
343 & 1100 \\
 & 1029 \\ \hline
3462 & 7100 \\
 & 6924 \\ \hline
\end{array}$$

$\therefore \sqrt{3} = 1.732$.

Ex. 12. ***If $\sqrt{3}$ = 1.732, find the value of $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ correct to 3 places of decimal.*** **(S.S.C. 2004)**

Sol. $\sqrt{192} - (1 / 2)\sqrt{48} - \sqrt{75} = \sqrt{64 * 3} - (1/2) \sqrt{16 * 3} - \sqrt{25 * 3}$

$= 8\sqrt{3} - (1/2) * 4\sqrt{3} - 5\sqrt{3}$

$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732$

Ex. 13. Evaluate: $\sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Sol. Given exp. = $\sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

$\therefore$ Given exp = $\sqrt{(95 * 85 * 18900) / (17 * 19 * 21)} = \sqrt{5 * 5 * 900} = 5 * 30 = 150$.

Ex. 14. ***Simplify*****:** $\sqrt{[(12.1)^2 - (8.1)^2] / [(0.25)^2 + (0.25)(19.95)]}$

Sol. Given exp. = $\sqrt{[(12.1 + 8.1)(12.1 - 8.1)] / [(0.25)(0.25 + 19.95)]}$

$= \sqrt{(20.2 * 4) /(0.25 * 20.2)} = \sqrt{4 / 0.25} = \sqrt{400 / 25} = \sqrt{16} = 4$.

Ex. 15. ***If x = 1 + $\sqrt{2}$ and y = 1 - $\sqrt{2}$, find the value of $(x^2 + y^2)$.***

Sol. $x^2 + y^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 2[(1)^2 + (\sqrt{2})^2] = 2 * 3 = 6.$

Ex. 16. ***Evaluate: √0.9 up to 3 places of decimal.***

Sol.

$$\begin{array}{r|l} 9 & 0.900000(0.948 \\ & \ \ 81 \\ \hline 184 & \ \ \ 900 \\ & \ \ \ 736 \\ \hline 1888 & \ \ \ 16400 \\ & \ \ \ 15104 \end{array}$$

$\therefore \sqrt{0.9} = 0.948$

Ex.17. If √15 = 3.88, find the value of √ (5/3).

Sol. $\sqrt{(5/3)} = \sqrt{(5 * 3) / (3 * 3)} = \sqrt{15} / 3 = 3.88 / 3 = 1.2933\ldots. = 1.29\overline{3}.$

Ex. 18. Find the least square number which is exactly divisible by 10,12,15 and 18.

Sol. L.C.M. of 10, 12, 15, 18 = 180. Now, $180 = 2 * 2 * 3 * 3 * 5 = 2^2 * 3^2 * 5$.

To make it a perfect square, it must be multiplied by 5.

$\therefore$ Required number = $(2^2 * 3^2 * 5^2) = 900$.

Ex. 19. Find the greatest number of five digits which is a perfect square.

(R.R.B. 1998)

Sol. Greatest number of 5 digits is 99999.

$$\begin{array}{r|l} 3 & 99999(316 \\ & 9 \\ \hline 61 & \ \ 99 \\ & \ \ 61 \\ \hline 626 & \ \ 3899 \\ & \ \ 3756 \\ \hline & \ \ \ \ 143 \end{array}$$

$\therefore$ Required number == (99999 - 143) = 99856.

Ex. 20. Find the smallest number that must be added to 1780 to make it a perfect square.

Sol.

$$\begin{array}{r|l} 4 & 1780\ (42 \\ & 16 \\ \hline 82 & \ \ 180 \\ & \ \ 164 \\ \hline & \ \ \ \ 16 \\ \hline \end{array}$$

$\therefore$ Number to be added = $(43)^2 - 1780 = 1849 - 1780 = 69$.

Ex. 21. √2 = 1.4142, find the value of √2 / (2 + √2).

Sol. $\sqrt{2} / (2 + \sqrt{2}) = \sqrt{2} / (2 + \sqrt{2}) * (2 - \sqrt{2}) / (2 - \sqrt{2}) = (2\sqrt{2} - 2) / (4 - 2)$

$= 2(\sqrt{2} - 1) / 2 = \sqrt{2} - 1 = 0.4142.$

22. If $x = (\sqrt{5} + \sqrt{3}) / (\sqrt{5} - \sqrt{3})$ and $y = (\sqrt{5} - \sqrt{3}) / (\sqrt{5} + \sqrt{3})$, find the value of $(x^2 + y^2)$.

Sol.

$x = [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} - \sqrt{3})] * [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} + \sqrt{3})] = (\sqrt{5} + \sqrt{3})^2 / (5 - 3)$

$=(5 + 3 + 2\sqrt{15}) / 2 = 4 + \sqrt{15}.$

$y = [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} + \sqrt{3})] * [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} - \sqrt{3})] = (\sqrt{5} - \sqrt{3})^2 / (5 - 3)$

$=(5 + 3 - 2\sqrt{15}) / 2 = 4 - \sqrt{15}.$

$\therefore \quad x^2 + y^2 = (4 + \sqrt{15})^2 + (4 - \sqrt{15})^2 = 2[(4)^2 + (\sqrt{15})^2] = 2 * 31 = 62.$

Ex. 23. Find the cube root of 2744.

Sol. **Method:** Resolve the given number as the product of prime factors and take the product of prime factors, choosing one out of three of the same prime factors. Resolving 2744 as the product of prime factors, we get:

2	2744
2	1372
2	686
7	343
7	49
	7

$2744 = 2^3 \times 7^3.$

$\therefore \quad \sqrt[3]{2744} = 2 \times 7 = 14.$

Ex. 24. By what least number 4320 be multiplied to obtain a number which is a perfect cube?

Sol. Clearly, $4320 = 2^3 * 3^3 * 2^2 * 5.$

To make it a perfect cube, it must be multiplied by $2 * 5^2$ i.e,50.

6.AVERAGE

Ex.1:Find the average of all prime numbers between 30 and 50?
Sol: there are five prime numbers between 30 and 50.
They are 31,37,41,43 and 47.
Therefore the required average=(31+37+41+43+47)/5 ⇔199/5 ⇔ 39.8.

Ex.2. find the average of first 40 natural numbers?
Sol: sum of first n natural numbers=n(n+1)/2;
So,sum of 40 natural numbers=(40*41)/2 ⇔820.
Therefore the required average=(820/40) ⇔20.5.

Ex.3. find the average of first 20 multiples of 7?
Sol: Required average =7(1+2+3+…….+20)/20 ⇔(7*20*21)/(20*2) ⇔(147/2)=73.5.

Ex.4. the average of four consecutive even numbers is 27. find the largest of these numbers?
Sol: let the numbers be x,x+2,x+4 andx+6. then,
(x+(x+2)+(x+4)+(x+6))/4) = 27
⇔(4x+12)/4 = 27
⇔x+3=27 ⇔ x=24.
Therefore the largest number=(x+6)=24+6=30.

Ex.5. there are two sections A and B of a class consisting of 36 and 44 students respectively. If the average weight of section A is 40kg and that of section B is 35kg, find the average weight of the whole class?
Sol: total weight of(36+44) students=(36*40+44*35)kg =2980kg.
Therefore weight of the total class=(2980/80)kg =37.25kg.

Ex:6.nine persons went to a hotel for taking their meals 8 of them spent Rs.12 each on their meals and the ninth spent Rs.8 more than the average expenditure of all the nine.What was the total money spent by them?
Sol: Let the average expenditure of all nine be Rs.x
Then 12*8+(x+8)=9x or 8x=104 or x=13.
Total money spent = 9x=Rs.(9*13)=Rs.117.

Ex.7: Of the three numbers, second is twice the first and is also thrice the third.If the average of the three numbers is 44.Find the largest number.
Sol: Let the third number be x.
Then second number = 3x.
First number=3x/2.
Therefore x+3x+(3x/2)=(44*3) or x=24
So largest number= 2^{nd} number=3x=72.

Ex.8:The average of 25 result is 18.The average of 1st 12 of them is 14 & that of last 12 is 17.Find the 13th result.
Sol: Clearly 13th result=(sum of 25 results)-(sum of 24 results)
=(18*25)-(14*12)+(17*12)
=450-(168+204)
=450-372
=78.

Ex.9:The Average of 11 results is 16, if the average of the 1st 6 results is 58 & that of the last 63. Find the 6th result.
Sol: 6th result = (58*6+63*6-60*11)=66

Ex.10:The average waight of A,B,C is 45 Kg. The avg wgt of A & B be 40Kg & that of B,C be 43Kg. Find the wgt of B.
Sol. Let A,B,c represent their individual wgts.
Then,
A+B+C=(45*3)Kg=135Kg
A+B=(40*2)Kg=80Kg & B+C=(43*2)Kg=86Kg
B=(A+B)+(B+C)-(A+B+C)
=(80+86-135)Kg
=31Kg.

Ex. 11. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by3 months. Find the age of the teacher.
Sol. Total age of 39 persons = (39 x 15) years
= 585 years.
Average age of 40 persons= 15 yrs 3 months
= 61/4 years.
Total age of 40 persons = ((61/4)x 40) years= 610 years.

:. Age of the teacher = (610 - 585) years=25 years.

Ex. 12. The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the new man.
Sol. Total weight increased =(1.8 x 10) kg =18 kg.
:. Weight of the new man =(53 + 18) kg =71 kg.

Ex. 13. There were 35 students in a hostel. Due to the admission of 7 new students, ;he expenses of the mess were increased by Rs. 42 per day while the average expenditure per head diminished by Rs 1. Wbat was the original expenditure of the mess?
Sol. Let the original average expenditure be Rs. x. Then,
42 (x - 1) - *35x*=42 ⇔ *7x*= 84 ⇔ x =12.
Original expenditure = Rs. (35 x 12) =Rs. 420. .

14. A batsman makes a score of 87 runs in the 17th inning and thus increases his avg by 3. Find his average after 17th inning.

Sol. Let the average after 17th inning = x.

Then, average after 16th inning = $(x - 3)$.

:. 16 (x - 3) + 87 = $17x$ or x = (87 - 48) = 39.

Ex.15. Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km per hour and returns back to A with a uniform speed of 56 km perhour. Find the average speed of the train during the whole journey.

Sol. Required average speed = ((2xy)/(x+y)) km / hr

=(2 x 84 x 56)/(84+56)km/hr

= (2*84*56)/140 km/hr

=67.2 km/hr.

7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyze the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex.1. A number is as much greater than 36 as is less than 86. Find the number.

Sol. Let the number be x. Then, $x - 36 = 86 - x \Rightarrow 2x = 86 + 36 = 122 \Rightarrow x = 61$.
Hence, the required number is 61.

Ex. 2. Find a number such that when 15 is subtracted from 7 times the number, the Result is 10 more than twice the number. (Hotel Management, 2002)

Sol. Let the number be x. Then, $7x - 15 = 2x + 10 \Rightarrow 5x = 25 \Rightarrow x = 5$.
Hence, the required number is 5.

Ex. 3. The sum of a rational number and its reciprocal is 13/6. Find the number. (S.S.C. 2000)

Sol. Let the number be x.
Then, $x + (1/x) = 13/6 \Rightarrow (x^2 + 1)/x = 13/6 \Rightarrow 6x^2 - 13x + 6 = 0$
$\Rightarrow 6x^2 - 9x - 4x + 6 = 0 \Rightarrow (3x - 2)(2x - 3) = 0$
$\Rightarrow x = 2/3$ or $x = 3/2$
Hence the required number is 2/3 or 3/2.

Ex. 4. The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.

Sol. Let the numbers be x and $(184 - x)$. Then,
$(X/3) - ((184 - x)/7) = 8 \Rightarrow 7x - 3(184 - x) = 168 \Rightarrow 10x = 720 \Rightarrow x = 72$.
So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.

Sol. Let the number be x and y. Then,
$x - y = 11$ ----(i) and $1/5\ (x + y) = 9 \Rightarrow x + y = 45$ ----(ii)
Adding (i) and (ii), we get: $2x = 56$ or $x = 28$. Putting $x = 28$ in (i), we get: $y = 17$.
Hence, the numbers are 28 and 17.

Ex. 6. If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers. (S.S.C. 2003)

Sol. Let the numbers be x and y. Then, $x + y = 42$ and $xy = 437$
$x - y = \text{sqrt}[(x + y)^2 - 4xy] = \text{sqrt}[(42)^2 - 4 \times 437] = \text{sqrt}[1764 - 1748] = \text{sqrt}[16] = 4$.
Required difference = 4.

Ex. 7. The sum of two numbers is 16 and the sum of their squares is 113. Find the

numbers.

Sol. Let the numbers be x and $(15 - x)$.

Then, $x^2 + (15 - x)^2 = 113$ => $x^2 + 225 + X^2 - 30x = 113$

=> $2x^2 - 30x + 112 = 0$ => $x^2 - 15x + 56 = 0$

=> $(x - 7)(x - 8) = 0$ => $x = 7$ or $x = 8$.

So, the numbers are 7 and 8.

Ex. 8. The average of four consecutive even numbers is 27. Find the largest of these numbers.

Sol. Let the four consecutive even numbers be x, x + 2, x + 4 and x + 6.

Then, sum of these numbers = (27 x 4) = 108.

So, $x + (x + 2) + (x + 4) + (x + 6) = 108$ or $4x = 96$ or $x = 24$.

:. Largest number = $(x + 6) = 30$.

Ex. 9. The sum of the squares of three consecutive odd numbers is 2531.Find the numbers.

Sol. Let the numbers be x, x + 2 and x + 4.

Then, $X^2 + (x + 2)^2 + (x + 4)^2 = 2531$ => $3x^2 + 12x - 2511 = 0$

=> $X^2 + 4x - 837 = 0$ => $(x - 27)(x + 31) = 0$ => $x = 27$.

Hence, the required numbers are 27, 29 and 31.

Ex. 10. Of two numbers, 4 times the smaller one is less then 3 times the 1arger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.

Sol. Let the numbers be x and *y,* such that x > y

Then, $3x - 4y = 5$ ...(i) and $(x + y) - 6(x - y) = 6$ => $-5x + 7y = 6$ …(ii)

Solving (i) and *(ii),* we get: x = 59 and $y = 43$.

Hence, the required numbers are 59 and 43.

Ex. 11. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1.If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?

Sol. Let the ten's digit be x. Then, unit's digit = $(x + 3)$.

Sum of the digits = $x + (x + 3) = 2x + 3$. Number = $10x + (x + 3) = 11x + 3$.

11x+3 / 2x + 3 = 4 / 1 => $11x + 3 = 4(2x + 3)$ => $3x = 9$ => $x = 3$.

Hence, required number = $11x + 3 = 36$.

Ex. 12. A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.

Sol. Let the ten's digit be x. Then, unit's digit = $(9 - x)$.

Number = $10x + (9 - x) = 9x + 9$.

Number obtained by reversing the digits = $10(9 - x) + x = 90 - 9x$.

therefore, $(9x + 9) - 63 = 90 - 9x$ => $18x = 144$ => $x = 8$.

So, ten's digit = 8 and unit's digit = 1.

Hence, the required number is 81.

Ex. 13. A fraction becomes 2/3 when 1 is added to both, its numerator and denominator.

And ,it becomes 1/2 when 1 is subtracted from both the numerator and denominator. Find

the fraction.

Sol. Let the required fraction be x/y. Then,

x+1 / y+1 = 2 / 3 => 3x – 2y = - 1 …(i) and x – 1 / y – 1 = 1 / 2

⇨ 2x – y = 1 …(ii)

Solving (i) and (ii), we get : x = 3 , y = 5

therefore, Required fraction= 3 / 5.

Ex. 14. 50 is divided into two parts such that the sum of their reciprocals is 1/ 12.Find the two parts.

Sol. Let the two parts be x and (50 - *x).*

Then, 1 / x + 1 / (50 – x) = 1 / 12 => (50 – x + x) / x (50 – x) = 1 / 12

=> x^2 – 50x + 600 = 0 => (x – 30) (x – 20) = 0 => x = 30 or x = 20.

So, the parts are 30 and 20.

Ex. 15. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers)

Sol. Let the numbers be x, *y* and z. Then,

x+ *y* = 10 *...(i)* *y* + z = 19 *...(ii)* x + z = 21 …(iii)

Adding (i) *,(ii)* and *(iii),* we get: *2 (x + y + z)* = 50 or (x + *y* + z) = 25.

Thus, x= (25 - 19) = 6; *y* = (25 - 21) = 4; z = (25 - 10) = 15.

Hence, the required numbers are 6, 4 and 15.

8. PROBLEMS ON AGES

Ex. 1. Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev ? (Hotel Management,2002)

Sol. Let Rajeev's present age be x years. Then,

Rajeev's age after 15 years = (x + 15) years.

Rajeev's age 5 years back = (x - 5) years.

:. x + 15 = 5 (x - 5) ⇔ x + 15 = 5x - 25 ⇔ 4x = 40 ⇔ x = 10.

Hence, Rajeev's present age = 10 years.

Ex. 2. ***The ages* of *two persons differ by* 16 *years.* If 6 *years ago, the* elder one be 3 *times as old as the younger* one, *find their present ages.*** (A.A.O. Exam,2003)

Sol. Let the age of the younger person be x years.

Then, age of the elder person = (x + 16) years.

:. 3 (x - 6) = (x + 16 - 6) ⇔ 3x -18 = x + 10 ⇔ 2x = 28 ⇔ x = 14.

Hence, their present ages are 14 years and 30 years.

Ex. 3. ***The product* of *the ages* of *Ankit and Nikita is 240.* If *twice the* age of Nikita *is more than Ankit's age by* 4 *years, what is Nikita's age?*** (S.B.I.P.O,1999)

Sol. Let Ankit's age be x years. Then, Nikita's age = 240/xyears.

$\therefore$ 2 × (240 /x) – x = 4 ⇔ 480 – x^2 = 4x ⇔ x^2 + 4x – 480 = 0

⇔ (x+24)(x-20) = 0 ⇔ x = 20.

Hence, Nikita's age = (22_0) years = 12 years. 1

Ex. 4. ***The present age* of a *father is* 3 *years more than three times the* age of his *son. Three years hence, father's age will be 10 years more than twice the* age *of the* son. *Find the present age* of *the* father.** . **(S.S.C, 2003)**

Sol. Let the son's present age be x years. Then, father's present age = *(3x* + 3) years

$\therefore$ *(3x* + 3 + 3) = 2 (x + 3) + 10 ⇔ 3x + 6 = 2x + 16 ⇔ x = 10.

Hence, father's present age = *(3x* + 3) = ((3 × 10) + 3) years = 33 years.

Ex. 5. R*ohit was* 4 *times as old as his son* 8 *years ago. After* 8 *years, Rohit will be twice as old as his son. What are their present ages?*

Sol. Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = 4x years.

Son's age after 8 years = (x + 8) + 8 = (x + 16) years.

Rohit's age after 8 years = *(4x* + 8) + 8 = *(4x*+ 16) years.

$\therefore$ 2 (x + 16) = 4x + 16 ⇔ 2x = 16 ⇔ x = 8.

Hence, son's 'present age = (x + 8) = 16 years.

Rohit's present age = *(4x* + 8) = 40 years.

Ex. 6. One ***year ago, the ratio*** **of** ***Gaurav's and Sachin's age was*** **6: 7** ***respectively. Four years hence, this ratio would become*** **7: 8.** ***How old is Sa chin*** **?**

(NABARD, 2002)

Sol:

. Let Gaurav's and Sachin's ages one year ago be 6x and 7x years respectively. Then, Gaurav's age 4 years hence = $(6x + 1) + 4 = (6x + 5)$ years.

Sachin's age 4 years hence = $(7x + 1) + 4 = (7x + 5)$ years.

$\frac{6x+5}{7x+5} = \frac{7}{8} \Leftrightarrow 8(6x+5) = 7(7x + 5) \Leftrightarrow 48x + 40 = 49x + 35 \Leftrightarrow x = 5.$

Hence, Sachin's present age = $(7x + 1) = 36$ years.

,**7.** ***Abhay's age after six years will be three-seventh*** **of** ***his fathers age. Ten years*** **ago the** ***ratio*** **of** ***their ages was 1*** **: 5.** ***What is Abhay's father's age*** **at** ***present?***

Sol. Let the ages of Abhay and his father 10 years ago be x and 5x years respectively. Then,

Abhay's age after 6 years = $(x + 10) + 6 = (x + 16)$ years.

Father's age after 6 years = $(5x + 10) + 6 = (5x + 16)$ years.

$\backslash x + 16) = \frac{3}{7}(5x + 16) \Leftrightarrow 7(x + 16) = 3(5x + 16) \Leftrightarrow 7x + 112 = 15x + 48$

$\Leftrightarrow 8x = 64 \Leftrightarrow x = 8.$

Hence, Abhay's father's present age = $(5x + 10) = 50$ years.

9. SURDS AND INDICES

I IMPORTANT FACTS AND FORMULAE I

1. LAWS OF INDICES:

(i) $a^m \times a^n = a^{m+n}$
(ii) $a^m / a^n = a^{m-n}$
(iii) $(a^m)^n = a^{mn}$
(iv) $(ab)^n = a^n b^n$
(v) $(a/b)^n = (a^n / b^n)$
(vi) $a^0 = 1$

2. SURDS: *Let a be a rational number and* n *be a positive integer such that* $a^{1/n} = {}^n\text{sqrt}(a)$ *is irrational. Then* ${}^n\text{sqrt}(a)$ is *called a surd of order* n.

3. LAWS OF SURDS:

(i) ${}^n\sqrt{a} = a^{1/2}$
(ii) ${}^n\sqrt{ab} = {}^n\sqrt{a} * {}^n\sqrt{b}$
(iii) ${}^n\sqrt{a/b} = {}^n\sqrt{a} \;/\; {}^n\sqrt{b}$
(iv) $({}^n\sqrt{a})^n = a$
(v) ${}^m\sqrt{({}^n\sqrt{(a)})} = {}^{mn}\sqrt{(a)}$
(vi) $({}^n\sqrt{a})^m = {}^n\sqrt{a^m}$

I SOLVED EXAMPLES

Ex. 1. Simplify : (i) $(27)^{2/3}$ (ii) $(1024)^{-4/5}$ (iii) $(8/125)^{-4/3}$

Sol . (i) $(27)^{2/3} = (3^3)^{2/3} = 3^{(3*(2/3))} = 3^2 = 9$
(ii) $(1024)^{-4/5} = (4^5)^{-4/5} = 4^{\{5*((-4)/5)\}} = 4^{-4} = 1/4^4 = 1/256$
(iii) $(8/125)^{-4/3} = \{(2/5)^3\}^{-4/3} = (2/5)^{\{3*(-4/3)\}} = (2/5)^{-4} = (5/2)^4 = 5^4/2^4 = 625/16$

Ex. 2. Evaluate: (i) $(.00032)^{3/5}$ *(ii)l* $(256)^{0.16} \times (16)^{0.18}$.

Sol. (i) $(0.00032)^{3/5} = (32/100000)^{3/5}. = (2^5/10^5)^{3/5} = \{(2/10)^5\}^{3/5} = (1/5)^{(5*3/5)} = (1/5)^3 = 1/125$
(ii) $(256)^{0.16} * (16)^{0.18} = \{(16)^2\}^{0.16} * (16)^{0.18} = (16)^{(2*0.16)} * (16)^{0.18}$
$=(16)^{0.32} * (16)^{0.18} = (16)^{(0.32+0.18)} = (16)^{0.5} = (16)^{1/2} = 4.$

Ex. 3. *What is the quotient when* $(x^{-1} - 1)$ *is divided by* $(x - 1)$?

Sol. $\dfrac{x^{-1} - 1}{x - 1} = \dfrac{(1/x) - 1}{x - 1} = \dfrac{1 - x}{x} * \dfrac{1}{(x - 1)} = \dfrac{-1}{x}$

Hence, the required quotient is $-1/x$

Ex. 4. If $2^{x-1} + 2^{x+1} = 1280$, *then find the value* of x.

Sol. $2^{x-1} + 2^{X+1} = 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280$

$$\Leftrightarrow 2^{x-1} = 1280 / 5 = 256 = 2^8 \Leftrightarrow x - 1 = 8 \Leftrightarrow x = 9.$$

Hence, x = 9.

Ex. 5. *Find* the value of $[5(8^{1/3} + 27^{1/3})^3]^{1/4}$

Sol. $[5(8^{1/3} + 27^{1/3})^3]^{1/4} = [5\{(2^3)^{1/3} + (3^3)^{1/3}\}^3]^{1/4} = [5\{(2^{3 * 1/3})^{1/3} + (3^{3 * 1/3})^{1/3}\}^3]^{1/4}$

$$= \{5(2+3)^3\}^{1/4} = (5 * 5^3)^{1/4} = 5^{(4 * 1/4)} = 5^1 = 5.$$

Ex. 6. Find the Value of $\{(16)^{3/2} + (16)^{-3/2}\}$

Sol. $[(16)^{3/2} + (16)^{-3/2} = (4^2)^{3/2} + (4^2)^{-3/2} = 4^{(2 * 3/2)} + 4^{\{2 * (-3/2)\}}$

$$= 4^3 + 4^{-3} = 4^3 + (1/4^3) = (64 + (1/64)) = 4097/64.$$

Ex. 7. *If* $(1/5)^{3y} = 0.008$, *then find the value of* $(0.25)^y$.

Sol. $(1/5)^{3y} = 0.008 = 8/1000 = 1/125 = (1/5)^3 \Leftrightarrow 3y = 3 \Leftrightarrow Y = 1.$

$\therefore (0.25)^y = (0.25)^1 = 0.25.$

Ex. 8. *Find the value* of $\dfrac{(243)^{n/5} \times 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\dfrac{(243)^{n/5}\ x3^{2n+l}}{(3^2)^n \times 3^{n-1}} = \dfrac{3^{(5 * n/5)} \times 3^{2n+l}}{3^{2n} \times 3^{n-1}} = \dfrac{3^n \times 3^{2n+1}}{32n \times 3^{n-l}}$

$= \dfrac{3^{n+(2n+1)}}{3^{2n+n-1}} = \dfrac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+l)-(3n-l)} = 3^2 = 9.$

Ex. 9. *Find the value Of* $(2^{1/4}-1)(2^{3/4}+2^{1/2}+2^{1/4}+1)$

Sol.

Putting $2^{1/4} = x$, we get :

$(2^{1/4}-1)\ (2^{3/4}+2^{1/2}+2^{1/4}+1)=(x-1)(x^3+x^2+x+1)$, *where* $x = 2^{1/4}$

$=(x-1)[x^2(x+1)+(x+1)]$

$=(x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1)$

$=(x^4-1) = [(2^{1/4})^4-1] = [2^{(1/4*4)} - 1] = (2-1) = 1.$

Ex. 10. Find the value of $\dfrac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$

Sol. $\dfrac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \dfrac{6^{2/3} \times (6^7)^{1/3}}{(6^6)^{1/3}} = \dfrac{6^{2/3} \times 6^{(7 * 1/3)}}{6^{(6 * 1/3)}} = \dfrac{6^{2/3} \times 6^{(7/3)}}{6^2}$

$=6^{2/3} \times 6^{((7/3)-2)} = 6^{2/3} \times 6^{1/3} = 6^1 = 6.$

Ex. 11. If $x= y^a$, $y=z^b$ *and* $z=x^c$,*then find the value of abc.*

Sol. $z^1= x^c =(y^a)^c$ *[since* $x= y^a$*]*

$=y^{(ac)} = (z^b)^{ac}$ *[since* $y=z^b$*]*

$=z^{b(ac)}= z^{abc}$

$\therefore$ $abc = 1.$

Ex. 12. Simplify $[(x^a / x^b)^\wedge(a^2+b^2+ab)] * [(x^b / x^c)^\wedge\ b^2+c^2+bc)] * [(x^c/x^a)^\wedge(c^2+a^2+ca)]$

Sol.

Given Expression

$= [\{x^{(o-b)}\}^\wedge(a^2 + b^2 + ob)].['(x^{(b-c)}\}^\wedge\ (b^2 + c^2 + bc)].['(x^{(c-a)}\}^\wedge(c^2 + a^2 + ca])$

$= [x^{(a-b)(a2+b2+ab)} . x^{(b-c)(b2+c2+bc)}.x^{(c-a)(c2+a2+ca)}]$

$= [x^\wedge(a^3-b^3)].[x^\wedge(b^3-e^3)].[x^\wedge(c^3-a^3)] = x^\wedge(a^3-b^3+b^3-c^3+c^3-a^3) = x^0 = 1.$

Ex. 13. *Which is larger* $\sqrt{2}$ or $\sqrt[3]{3}$?

Sol. Given surds are of order 2 and 3. Their L.C.M. is 6. Changing each to a surd of order 6, we get:

$$\sqrt{2} = 2^{1/2} = 2^{((1/2)*(3/2))} = 2^{3/6} = 8^{1/6} = \sqrt[6]{8}$$
$$\sqrt[3]{3} = 3^{1/3} = 3^{((1/3)*(2/2))} = 3^{2/6} = (3^2)^{1/6} = (9)^{1/6} = \sqrt[6]{9}.$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$ and hence $\sqrt[3]{3} > \sqrt{2}$.

Ex. 14. *Find the largest from among* $4\sqrt{6}$, $\sqrt{2}$ *and* $\sqrt[3]{4}$.

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C,M, is 12, Changing each to a surd of order 12, we get:

$\sqrt[4]{6} = 6^{1/4} = 6^{((1/4)*(3/3))} = 6^{3/12} = (6^3)^{1/12} = (216)^{1/12.}$

$\sqrt{2} = 2^{1/2} = 2^{((1/2)*(6/6))} = 2^{6/12} = (2^6)^{1/12} = (64)^{1/12}$.

$\sqrt[3]{4} = 4^{1/3} = 4^{((1/3)*(4/4))} = 4^{4/12} = (4^4)^{1/12} = (256)^{1/12}$.

Clearly, $(256)^{1/12} > (216)^{1/12} > (64)^{1/12}$

Largest one is $(256)^{1/12}$. i.e. $\sqrt[3]{4}$.

10.PERCENTAGE

IMPORTANT FACTS AND FORMULAE

1. **Concept of Percentage :** By a certain percent ,we mean that many hundredths. Thus x percent means x hundredths, written as x%.

   To express x% as a fraction : We have , x% = x/100.

   Thus, 20% =20/100 =1/5; 48% =48/100 =12/25, etc.

   To express a/b as a percent : We have, a/b =((a/b)*100)%.

   Thus, ¼ =[(1/4)*100] = 25%; 0.6 =6/10 =3/5 =[(3/5)*100]% =60%.

2. If the price of a commodity increases by R%, then the reduction in consumption so asnot to increase the expenditure is

   [R/(100+R))*100]%.

   If the price of the commodity decreases by R%,then the increase in consumption so as to decrease the expenditure is

   [(R/(100-R)*100]%.

3. **Results on Population :** Let the population of the town be P now and suppose it increases at the rate of R% per annum, then :

   1. Population after nyeras = P [1+(R/100)]^n.
   2. Population n years ago = P /[1+(R/100)]^n.

4. **Results on Depreciation :** Let the present value of a machine be P. Suppose it depreciates at the rate R% per annum. Then,

1. Value of the machine after n years = $P[1-(R/100)]^n$.
2. Value of the machine n years ago = $P/[1-(R/100)]^n$.

5. If A is R% more than B, then B is less than A by

[(R/(100+R))*100]%.

If A is R% less than B , then B is more than A by

[(R/(100-R))*100]%.

SOLVED EXAMPLES

Ex. 1. Express each of the following as a fraction :

(i) 56% **(ii) 4%** **(iii) 0.6%** **(iv) 0.008%**

sol. (i) 56% = 56/100= 14/25. (ii) 4% =4/100 =1/25.
(iii) 0.6 =6/1000 = 3/500. (iv) 0.008 = 8/100 = 1/1250.

Ex. 2. Express each of the following as a Decimal :

(i) 6% **(ii)28%** **(iii) 0.2%** **(iv) 0.04%**

Sol. (i) 6% = 6/100 =0.06. (ii) 28% = 28/100 =0.28.
(iii) 0.2% =0.2/100 = 0.002. (iv) 0.04%= 0.04/100 =0.004.

Ex. 3. Express each of the following as rate percent :

(i) 23/36 **(ii) 6 ¾** **(iii) 0.004**

Sol. (i) 23/36 = [(23/36)*100]% = [575/9]% = 63 8/9%.

(ii) 0.004 = [(4/1000)*100]% = 0.4%.

(iii) 6 ¾ =27/4 =[(27/4)*100]% = 675%.

Ex. 4. Evaluate :

(i) 28% of 450+ 45% of 280

(ii) 16 2/3% of 600 gm- 33 1/3% of 180 gm

Sol. (i) 28% of 450 + 45% of 280 =[(28/100)*450 + (45/100)*280] = (126+126) =252.

(iii) 16 2/3% of 600 gm –33 1/3% of 180 gm = [((50/3)*(1/100)*600) – ((100/3)*(1/3)*280)]gm = (100-60) gm = 40gm.

Ex. 5.
(i) 2 is what percent of 50 ?
(ii) ½ is what percent of 1/3 ?
(iii)What percent of 8 is 64 ?
(iv)What percent of 2 metric tones is 40 quintals ?
(v)What percent of 6.5 litres is 130 ml?

Sol.

(i) Required Percentage = [(2/50)*100]% = 4%.

(ii) Required Percentage = [(1/2)*(3/1)*100]% = 150%.

(iii)Required Percentage = [(84/7)*100]% = 1200%.

(iv) 1 metric tonne = 10 quintals.

Required percentage = [(40/(2 * 10)) * 100]% = 200%.

(v) Required Percentage = [(130/(6.5 * 1000)) * 100]% = 2%.

Ex. 6.
Find the missing figures :

(i) ?% of 25 = 20125 (ii) 9% of ? = 63 (iii) 0.25% of ? = 0.04

Sol.

(i) Let x% of 25 = 2.125. Then , (x/100)*25 = 2.125
X = (2.125 * 4) = 8.5.

(ii) Let 9% of x =6.3. Then , 9*x/100 = 6.3
X = [(6.3*100)/9] =70.

(iii) Let 0.25% of x = 0.04. Then , 0.25*x/100 = 0.04
X= [(0.04*100)/0.25] = 16.

Ex. 7.
Which is greatest in 16 (2/3) %, 2/5 and 0.17 ?

Sol. 16 (2/3)% =[(50/3)*)1/100)] = 1/6 = 0.166, 2/15 = 0.133. Clearly, 0.17 is the greatest.

Ex. 8.
If the sales tax reduced from 3 1/2 % to 3 1/3%, then what difference does it make to a person who purchases an article with market price of Rs. 8400 ?

Sol. Required difference = [3 ½ % of Rs.8400] – [3 1/3 % of Rs.8400]

= [(7/20-(10/3)]% of Rs.8400 =1/6 % of Rs.8400

= Rs. [(1/6)8(1/100)*8400] = Rs. 14.

Ex. 9. An inspector rejects 0.08% of the meters as defective. How many will be examine to project ?

Sol. Let the number of meters to be examined be x.

Then, 0.08% of x =2

[(8/100)*(1/100)*x] = 2

x = [(2*100*100)/8] = 2500.

Ex. 10. Sixty five percent of a number is 21 less than four fifth of that number. What is the number ?

Sol. Let the number be x.

Then, 4*x/5 –(65% of x) = 21

4x/5 –65x/100 = 21

5 x = 2100

x = 140.

Ex.11. Difference of two numbers is 1660. If 7.5% of the number is 12.5% of the other number , find the number ?

Sol. Let the numbers be x and y. Then , 7.5 % of x =12.5% of y

X = 125*y/75 = 5*y/3.

Now, x-y =1660

5*y/3 –y =1660

2*y/3= 1660

y =[(1660*3)/2] =2490.

One number = 2490, Second number =5*y/3 =4150.

Ex. 12.

In expressing a length 810472 km as nearly as possible with three significant digits , find the percentage error.

Sol. Error = (81.5 – 81.472)km = 0.028.

Required percentage = [(0.028/81.472)*100]% = 0.034%.

Ex. 13.

In an election between two candidates, 75% of the voters cast thier thier votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of votes enrolled in that election.

Sol.

Let the number of votes enrolled be x. Then ,

Number of votes cast =75% of x. Valid votes = 98% of (75% of x).

75% of (98% of (75%of x)) =9261.

[(75/100)*(98/100)*(75/100)*x] =9261.

X = [(9261*100*100*100)/(75*98*75)] =16800.

Ex.14. Shobha's mathematics test had 75 problems i.e.10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic ,40% of the algebra, and 60% of the geometry problems correctly. she did not pass the test because she got less than 60% of the problems right. How many more questions she would have to answer correctly to earn 60% of the passing grade?

Sol. Number of questions attempted correctly=(70% of 10 + 40% of 30 + 60% 0f 35)
=7 + 12+21= 45

questions to be answered correctly for 60% grade=60% of 75 = 45 therefore required number of questions= (45-40) = 5.

Ex.15. if 50% of (x-y) = 30% of (x+y) then what percent of x is y?

Sol.50% of (x-y)=30% of(x+y) ⇔ (50/100)(x-y)=(30/100)(x+y)
⇔5(x-y)=3(x+y) ⇔ 2x=8y ⇔ x=4y
therefore required percentage =((y/x) X 100)% = ((y/4y) X 100) =25%

Ex.16. Mr.Jones gave 40% of the money he had to his wife. he also gave 20% of the remaining amount to his 3 sons. half of the amount now left was spent on miscellaneous items and the remaining amount of Rs.12000 was deposited in the bank. how much money did Mr.jones have initially?

Sol. Let the initial amount with Mr.jones be Rs.x then,
Money given to wife= Rs.(40/100)x=Rs.2x/5.Balance=Rs(x-(2x/5)=Rs.3x/5.
Money given to 3 sons= Rs(3X((20/200) X (3x/5)) = Rs.9x/5.
Balance = Rs.((3x/5) – (9x/25))=Rs.6x/25.
Amount deposited in bank= Rs(1/2 X 6x/25)=Rs.3x/25.

Therefore 3x/25=12000 ⇔ x= ((12000 x 35)/3)=100000

So Mr.Jones initially had Rs.1,00,000 with him.

Short-cut Method : Let the initial amount with Mr.Jones be Rs.x

Then,(1/2)[100-(3*20)]% of x=12000

⇔ (1/2)*(40/100)*(60/100)*x=12000
⇔x=((12000*25)/3)=100000

Ex 17 10% of the inhabitants of village having died of cholera.,a panic set in , during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.
Sol:
Let the total number of orginal inhabitants be x.
((75/100))*(90/100)*x)=4050 ⇔ (27/40)*x=4050

⇔x=((4050*40)/27)=6000.

Ex.18 A salesman`s commission is 5% on all sales upto Rs.10,000 and 4% on all sales exceeding this.He remits Rs.31,100 to his parent company after deducing his commission . Find the total sales.
Sol:
Let his total sales be Rs.x.Now(Total sales) – (Commission)=Rs.31,100
x-[(5% of 10000 + 4% of (x-10000)]=31,100
x-[((5/100)*10000 + (4/100)*(x-10000)]=31,100

⇔x-500-((x-10000)/25)=31,100
⇔x-(x/25)=31200 ⇔ 24x/25=31200⇔x=[(31200*25)/24)=32,500.
Total sales=Rs.32,500

Ex .19 Raman`s salary was decreased by 50% and subsequently increased by 50%.How much percent does he lose?
Sol:
Let the original salary = Rs.100
New final salary=150% of (50% of Rs.100)=
Rs.((150/100)*(50/100)*100)=Rs.75.
Decrease = 25%

Ex.20 Paulson spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%.Find the percentage increase in his savings .
Sol:
Let the original income=Rs.100 . Then , expenditure=Rs.75 and savings =Rs.25
New income =Rs.120 , New expenditure =
Rs.((110/100)*75)=Rs.165/2
New savings = Rs.(120-(165/2)) = Rs.75/2
Increase in savings = Rs.((75/2)-25)=Rs.25/2
Increase %= ((25/2)*(1/25)*100)% = 50%.

Ex21. The salary of a person was reduced by 10% .By what percent should his reduced salary be raised so as to bring it at par with his original salary ?
Sol:
Let the original salary be Rs.100 . New salary = Rs.90.
Increase on 90=10 , Increase on 100=((10/90)*100)%
= (100/9)%

Ex.22 When the price fo a product was decreased by 10% , the number sold increased by 30%. What was the effect on the total revenue ?
Sol:
Let the price of the product be Rs.100 and let original sale be 100 pieces.
Then , Total Revenue = Rs.(100*100)=Rs.10000.
New revenue = Rs.(90*130)=Rs.11700.
Increase in revenue = ((1700/10000)*100)%=17%.

Ex 23 . If the numerator of a fraction be increased by 15% and its denominator be diminished by 8% , the value of the fraction is 15/16. Find the original fraction.
Sol:
Let the original fraction be x/y.
Then (115%of x)/(92% of y)=15/16 => (115x/92y)=15/16
⇨ ((15/16)*(92/115))=3/4

Ex.24 In the new budget , the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase ?

Sol:

Reduction in consumption = [((R/(100+R))*100]%

⇨ [(25/125)*100]%=20%.

Ex.25 The population of a town is 1,76,400 . If it increases at the rate of 5% per annum , what will be its population 2 years hence ? What was it 2 years ago ?

Sol:

Population after 2 years = 176400*[1+(5/100)]^2

=[176400*(21/20)*(21/40)]

= 194481.

Population 2 years ago = 176400/[1+(5/100)]^2

=[716400*(20/21)*(20/21)]= 160000.

Ex.26 The value of a machine depreiates at the rate of 10% per annum. If its present is Rs.1,62,000 what will be its worth after 2 years ? What was the value of the machine 2 years ago ?

Sol.

Value of the machine after 2 years

=Rs.[162000*(1-(10/100))^2] = Rs.[162000*(9/10)*(9/10)]

=Rs. 131220

Value of the machine 2 years ago

= Rs.[162000/(1-(10/100)^2)]=Rs.[162000*(10/9)*(10/9)]=Rs.200000

Ex27. During one year, the population of town increased by 5% . If the total population is 9975 at the end of the second year , then what was the population size in the beginning of the first year ?

Sol:

Population in the beginning of the first year

= 9975/[1+(5/100)]*[1-(5/100)] = [9975*(20/21)*(20/19)]=10000.

Ex.28 If A earns 99/3% more than B,how much percent does B earn less then A ?

Sol:

Required Percentage = [((100/3)*100)/[100+(100/3)]]%

=[(100/400)*100]%=25%

Ex. 29 If A`s salary is 20% less then B`s salary , by how much percent is B`s salary more than A`s ?

Sol:

Required percentage = [(20*100)/(100-20)]%=25%.

Ex30 .How many kg of pure salt must be added to 30kg of 2% solution of salt and water to increase it to 10% solution ?
Sol:
Amount of salt in 30kg solution = [(20/100)*30]kg=0.6kg
Let x kg of pure salt be added
Then , (0.6+x)/(30+x)=10/100⇔60+100x=300+10x
⇔90x=240 ⇔ x=8/3.

Ex 31. Due to reduction of 25/4% in the price of sugar , a man is able to buy 1kg more for Rs.120. Find the original and reduced rate of sugar.
Sol:
Let the original rate be Rs.x per kg.
Reduced rate = Rs.[(100-(25/4))*(1/100)*x}]=Rs.15x/16per kg
120/(15x/16)-(120/x)=1 ⇔ (128/x)-(120/x)=1
⇔ x=8.
So, the original rate = Rs.8 per kg
Reduce rate = Rs.[(15/16)*8]per kg = Rs.7.50 per kg

Ex.32 In an examination , 35% of total students failed in Hindi , 45% failed in English and 20% in both . Find the percentage of those who passed in both subjects .
Sol:
Let A and B be the sets of students who failed in Hindi and English respectively .
Then , n(A) = 35 , n(B)=45 , n(A∩B)=20.
So , n(A∪B)=n(A)+n(B)- n(A∩B)=35+45-20=60.
Percentage failed in Hindi and English or both=60%
Hence , percentage passed = (100-60)%=40%

Ex33. In an examination , 80% of the students passed in English , 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects , find the total number of students.
Sol:
Let the total number of students be x .
Let A and B represent the sets of students who passed in English and Mathematics respectively .
Then , number of students passed in one or both the subjects
= n(A∪B)=n(A)+n(B)- n(A∩B)=80% of x + 85% of x –75% of x
=[(80/100)x+(85/100)x-(75/100)x]=(90/100)x=(9/10)x
Students who failed in both the subjects = [x-(9x/10)]=x/10.
So, x/10=40 of x=400 .
Hence ,total number of students = 400.

11. PROFIT AND LOSS

IMPORTANT FACTS

COST PRICE: THE PRICE AT WHICH ARTICLE IS PURCHASED.ABBREVATED AS C.P.

SELLING PRICE: THE PRICE AT WHICH ARTICLE IS SOLD.

PROFIT OR GAIN:IF SP IS GREATER THAN CP,THE SELLING PRICE IS SAID TO HAVE PROFIT OR GAIN.

LOSS: IF SPIS LESS THAN CP,THE SELLER IS SAID TO INCURED A LOSS.

FORMULA

1.GAIN=(SP)-(CP). 2.LOSS=(CP)-(SP).
3.LOSS OR GAIN IS ALWAYS RECKONED ON CP
4. GAIN %={GAIN*100}/CP.
5.LOSS%={LOSS*100}/CP.
6.SP={(100+GAIN%) /100}*CP.
7.SP={(100-LOSS%)/100}*CP.
8.{100/(100+GAIN%)} *SP
9.CP=100/(100-LOSS%)}*SP
10.IF THE ARTICLE IS SOLD AT A GAIN OF SAY 35%, THEN SP =135% OF CP
11.IF A ARTICLE IS SOLD AT A LOSS OF SAY 35%. THEN SP=65% OF CP.
12.WHEN A PERSON SELLS TWO ITEMS,ONE AT A GAIN OF X% AND OTHER AT A LOSS OF X%.THEN THE SELLER ALWAYS INCURES A LOSS GIVEN:
{LOSS%=(COMON LOSS AND GAIN)^2}/10.=(X/10)^2
13.IF THE TRADER PROFESSES TO SELL HIS GOODS AT CP BUT USES FALSE WEIGHTS,THEN
GAIN=[ERROR/(TRUE VALUE)-(ERROR)*100]%

SOLVED PROBLEMS

ex.1 A man buys an article for rs.27.50 and sells it for rs.28.50. find his gain %.

sol. cp=rs27.50, sp=rs 28.50

gain=rs(28.50 –27.50)=rs1.10

so gain%={(1.10/27.50)*100}=4%

Ex.2. If the a radio is sold for rs 490 and sold for rs 465.50.find loss%.

sol. cp=rs490,sp= 465.50.

loss=rs(490-465.50)=rs 24.50.

loss%=[(24.50/490)*100]%=5%

Ex.3.find S.P when

(i)CP=56.25,gain=20%.

sol.

(i)SP =20% of rs 56.25 ,=rs{(120/100)*56.25}=rs67.50.

(ii)CP=rs 80.40,loss=5%

sol: sp=85% of rs 80.40

=rs {(85/100)*80.40}=rs 68.34.

ex.4 find cp when:

(i) sp =rs 40.60 : gain=16%

(ii) sp=rs51.70:loss=12%

(i) cp=rs{(100/116)*40.60}=rs 35.

(ii) cp=rs{(100/88)*51.87}=rs58.75.

ex.5 A person incures loss for by selling a watch for rs1140.at what price should the watch be sold to earn a 5% profit ?

sol. let the new sp be rsx.then

(100-loss%) : (1^{st} sp)=(100+gain%) <u>(2^{nd} sp)</u>

⇨ {(100-5)/1400}={(100+5)/x}=> x={(105*1140)/95} =1260.

⇨

ex.6 A book was sold for rs 27.50 with a profit of 10%. if it were sold for rs25.75, then what would be % of profit or loss?

sol. SP=rs 27.50: profit =10%.

sol. CP=rs {(100/110)*27.50}=rs 25.

When sp =Rs25.75 ,profit =Rs(25.75-25)=Rs 0.75

Profit%={(0.75/25)*100}%=25/6%=3%

Ex7 .If the cost price is 96% of sp then whqt is the profit %

Sol. sp=Rs100 : then cp=Rs 96:profit =Rs 4.

Profit={(4/96)*100}%=4.17%

Ex.8. The cp of 21 articles is equal to sp of 18 articles.find gain or loss %

CP of each article be Rs 1

CP of 18 articles =Rs18 ,sp of 18 articles =Rs 21.

Gain%=[(3/18)*100]%=50/3%

Ex.9 By selling 33 metres of cloth , one gains the selling price of 11 metres . Find the gain percent .

Sol:

(SP of 33m)-(CP of 33m)=Gain=SP of 11m

SP of 22m = CP of 33m

Let CP of each metre be Re.1 , Then, CP of 22m= Rs.22,SP of 22m=Rs.33.

Gain%=[(11/22)*100]%=50%

Ex10 A vendor bought bananas at 6 for Rs.10 and sold them at Rs.4 for Rs.6 .Find his gain or loss percent .

Sol:

Suppose , number of bananas bought = LCM of 6 and 4=12

CP=Rs.[(10/6)*12]=Rs.20 ; SP= Rs[(6/4)*12]=Rs.18

Loss%=[(2/20)*100]%=10%

Ex.11. A man brought toffees at for a rupee. How many for a rupee must he sell to gain 50%?

Sol. C.P of 3 toffees=Re 1; S.P of 3 toffees =150% of Re.1=3/2.

For Rs.3/2, toffees sold =3, for Re.1, toffees sold = [3*(2/3)] = 2.

Ex. 12.A grocer purchased 80 kg of sugar at Rs.13.50 per kg and mixed it with 120kg sugar at Rs.16per kg. At what rate should he sell the mixer to gain 16%?

Sol .C.P of 200 kg of mixture = Rs. (80 * 13.50+120*16) = Rs.3000.

S.P =116% 0f Rs.3000 =Rs.[(116/200) *3000]=Rs.3480.

∴ Rate of S.P of the mixture =Rs.[3480/200] per kg =Rs.17.40 per kg.

Ex.13. Pure ghee cost Rs.100 per kg. After adulterating it with vegetable oil costing Rs.50 per kg, A shopkeeper sells the mixture at the rate of Rs.96 per kg, thereby making a profit of 20%.In What ratio does he mix the two?

Sol. Mean cost price =Rs. [(100/120)*96] =Rs.80 per kg.

By the rate of allegation :

C.P of 1kg ghee   C.P of 1kg oil

100   50

Mean price

80

30   20

$\therefore$ Required ratio =30:20 =3:2.

Ex. 14. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg weight . Find his gain percent.

Sol .Gain% $= \left[\frac{\text{Error}}{\text{(error value)-(error)}} *100 \right]\% = [(40/960)*100]\ \% = 4\frac{1}{6}\ \%$

Ex 15. If the manufacturer gains 10%,the wholesale dealer 15% and the retailer 25% ,then find the cost of production of a ,the retail price of which is Rs.1265?

Sol:

Let the cost of production of the table be Rs x

The ,125% of 115% of 110% of x=1265

⇨ 125/100*115/100*110/100*x=1265=>253/160*x=>1265=>x=(1265 *160/253)=Rs.800

Ex16 . Monika purchased a pressure cooker at 9/10th of its selling price and sold it at 8% more than its S.P .find her gain percent.

Sol:

Let the s.p be Rs. X .then C.P = Rs.9x/10,Receipt=108% of rs.x=Rs 27x/25

Gain=Rs (27x/25*9x/10)=Rs(108x-90x/100)=Rs18x/100

Gain%=(18x/100*10/9x*100)%=20%

Ex .17 An article is sold at certain price. By selling it at 2/3 of its price one losses 10%,find the gain at original price ?

sol:

let the original s.p be Rs x. then now S.P=Rs2x/3,loss=10%

now C.P=Rs20x/27*27/20x*100)%=35%

Ex .18. A tradesman sold an article at a loss of 20%.if the selling price has been increased by Rs100,ther would have been a gain of 5%.what was the cost price of the article?

Sol:

Let C.P be Rs x. then (105% of x)-(80 % of x)=100 or 25% of x=100

⇨ x/4=100 or x=400

⇨ so,C.P =Rs 400

Ex 19. A man sells an article at a profit of 25%if he had bought it 20% less and sold it for Rs 10.50 less,he would have gained 30%find the cost price of the article.

Sol:

Let the C.P be Rs x

1^{st} S.P=125% of x =125x/100=5x/4;2^{nd} S.P=80% of x=80x/100=4x/5

2^{nd} S.P=130% of 4x/5=(130/100*4x/5)=26x/25

=>5x/4-26x/25=10.50⇔x=(10.50*100)/21=50

hence C.P=Rs.50

Ex 20.The price of the jewel,passing through three hands,rises on the whole by65%.if the first and the second sellers 20%and25% profit respectively find the percentage profit earned by the third seller.

Sol:

Let the orginal price of the jewel be Rs p and let the profit earned by the thrid seller be x%

Then,(100+x)% of 125% OF 120% OF P=165% OF P

⇨ ((100+X)/100*125/100*120/100*P)=(165/100*P)

⇨ (100+X)=(165*100*100)/(125*120)=110=>X=10%

Ex21 . A man 2 flats for Rs 675958 each.on one he gains 16% while on the other he losses 16%. How much does he gain/loss in the whole transaction?

Sol:

In this case there will be alwalys loss. The selling price is immaterial

Hence, loss % = (common loss and gain%)^2 /10=(16/10)%=(64/25)%=2.56%

Ex.22. A dealer sold three-fourth of his article at a gain of 20% and remaining at a cost price. Find the gain earned by him at the two transcation.

Sol:

Let the C.P of the whole be Rs x

C.P of $3/4^{th}$ =Rs 3x/4,C.P of $1/4^{th}$ =Rs x/4

⇨ total S.P=Rs [(120%of 3x/4)+x/4]=Rs(9x/10+x/4)=Rs 23x/20

⇨ gain=Rs(23x/20-x)=Rs 3x/20

⇨ gain%=3x/20*1/x*100)%=15%

Ex 23 ..A man bought a horse and a car riage for Rs 3000.he sold the horse at a gain of 20% and the carriage at a loss of 10%,thereby gaining 2% on the whole.find the cost of the horse.

Sol:

Let the C.p of the horse be Rs.x, then C.P of the carriage =Rs(3000-x)

20% of x-10% of(3000-x)=2% of 3000

⇨ x/5-(3000-x)/10=60=.2x-3000+x=600=.3x+3600=>x=1200

⇨ hence,C.P of the horse =Rs 1200

Ex 24 find the single discount equivalent to a series discount of 20% ,10% and 5%'

sol:

let the marked price be Rs 100

then ,net S.P=95% of 90% of 80% of Rs 100

=Rs(95/100*90/100*80/100*100)=Rs68.40

Ex .25 After getting 2 successive discounts, a shirt with a list price of Rs 150 is available at Rs 105. If the second discount is 12.55,find the first discount.

Sol:

Let the first discount be x%

Then,87.5% of (100-x)% of 150= 105

- ⇨ 87.5/100*(100-x)/100*450=150=>105=>100-x=(105*100*100)/(150*87.5)=80
- ⇨ x=(100-80)=20
- ⇨ first discount = 20%

Ex .26 An uneducated retailar marks all its goods at 50% above the cost price and thinking that he will still make 25% profit,offers a discount of 25% on the marked price.what is the actual profit on the sales?

Sol:

Let C.P =Rs 100.then ,marked price =Rs100

S.P=75% of Rs 150=Rs112.50

Hence,gain%=12.50%

Ex27 .A retailer buys 40 pens at the market price of 36 pens from a wholesaler ,if he sells these pens giving a discount of 1% ,what is the profit % ?

sol:

let the market price of each pen be Rs 1

then,C.P of 40 pens = Rs 36 S.P of 40 pens =99% of Rs 40=Rs 39.60

profit %=((3.60*100)/36) %=10%

Ex 28 . At what % above C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?

Sol

Let C.P be Rs 100.then S.P be Rs 133

Let the market price be Rs x

Then 90% of x=133=>95x/100=133=>x=(133*100/95)=140

Market price = 40% above C.P

Ex .29 . When a producer allows 36% commission on retail price of his product, he earns a profit of 8.8%. what would be his profit % if the commision is reduced by 24%?

Sol:

Let the retail price =Rs 100.then, commission=Rs 36

S.P=Rs(100-36)=Rs 64

But, profit=8.8%

C.P=Rs(100/108.8*64)=Rs 1000/17

New commission =Rs12. New S.P=Rs(100-12)Rs 88

Gain=Rs(88-1000/17)=Rs 496/17

Gain%=(496/17*17/1000*100)%=49.6%

12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

I. RATIO: The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as a:b. In the ratio a:b, we call a as the **first term or antecedent** and b, the **second term or consequent.**

Ex. The ratio 5: 9 represents 5/9 with antecedent = 5, consequent = 9.
Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
Ex. 4: 5 = 8: 10 = 12: 15 etc. Also, 4: 6 = 2: 3.

2. **PROPORTION:** ***The equality of two ratios is called proportion.***
If *a:* b = c: d, we write, *a:* b:: c : d and we say that *a,* b, c, d are in proportion . Here *a* and d are called extremes, while b and c are called mean terms.
Product of means = Product of extremes.
Thus, a: b:: c : d <=> (b x c) = (a x d).

3. (i) Fourth Proportional: If *a* : b = c: d, then d is called the fourth proportional to *a,* b, c.
(ii) Third Proportional: If *a*: b = b: c, then c is called the third proportional to a and b.
(iii) Mean Proportional: Mean proportional between *a* and *b* is *square root of ab*

4. (i) COMPARISON OF RATIOS:
We say that *(a*: *b)* > (c: *d)* <=> (a/b)>(c /d).
(ii) COMPOUNDED RATIO:
The compounded ratio of the ratios (a: *b),* (c: *d),* (e : f) is *(ace*: bdf)

5. (i) *Duplicate ratio* of *(a* : b) is $(a^2 : b^2)$.
(ii) Sub-duplicate ratio of (a : *b)* is $(\sqrt{a} : \sqrt{b})$.
(iii)Triplicate ratio of (a : *b)* is $(a^3 : b^3)$.
(iv) Sub-triplicate ratio of (a : *b)* is $(a^{1/3} : b^{1/3})$.
(v) If (a/b)=(c/d), then ((a+b)/(a-b))=((c+d)/(c-d)) **(Componendo and dividendo)**

6. VARIATION:
(i) We say that x is directly proportional to *y,* if x = *ky* for some constant k and we write, $x \propto y$.
(ii) We say that x is inversely proportional to *y,* if xy = *k* for some constant k and we write, x∞(1/y)

SOLVED PROBLEMS

Ex. 1. ***If*** **a : *b* = 5 : 9 *and* b : c = 4: 7, *find* a : b : c.**

Sol. a:b=5:9 and b:c=4:7= (4X9/4): (7x9/4) = 9:63/4
a:b:c = 5:9:63/4 =20:36:63.

Ex. 2. ***Find:***

(i) *the fourth proportional* to 4, 9, 12;
(ii) *the third proportional* to 16 *and* 36;
iii) the* mean *proportional between 0.08 and 0.18.

Sol.

i) Let the fourth proportional to 4, 9, 12 be x.

Then, 4 : 9 : : 12 : x ⇔4 x x=9x12 ⇔ X=(9 x 12)/14=27;
Fourth proportional to 4, 9, 12 is 27.

(ii) Let the third proportional to 16 and 36 be x.

Then, 16 : 36 : : 36 : x ⇔16 x *x* = 36 x 36 ⇔ x=(36 x 36)/16 =81
Third proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18
√0.08 x 0.18 =√8/100 x 18/100= √144/(100 x 100)=12/100=0.12

Ex. 3. ***If*** **x : *y* = 3 : 4, *find* (4x + *5y)* : (5x - *2y).***

Sol. X/Y=3/4 ⇔ (4x+5y)/(5x+2y)= (4(x/y)+5)/(5 (x/y)-2) =(4(3/4)+5)/(5(3/4)-2)

=(3+5)/(7/4)=32/7

Ex. 4. ***Divide Rs.* 672 *in the ratio* 5 : 3**.

Sol. Sum of ratio terms = (5 + 3) = 8.

First part = Rs. (672 x (5/8)) = Rs. 420; Second part = Rs. (672 x (3/8)) = Rs. 252.

Ex. 5. *Divide Rs.* 1162 among *A, B,* C *in the ratio* 35 : 28 : *20.*

Sol. Sum of ratio terms = (35 + 28 + 20) = 83.

A's share = Rs. (1162 x (35/83))= Rs. 490; B's share = Rs. (1162 x(28/83))= Rs. 392;

C's share = Rs. (1162 x (20/83))= Rs. 280.

Ex. 6. *A bag contains* 50 *p,* 25 *P and* 10 *p coins in the ratio* 5: 9: 4, *amounting* to *Rs. 206. Find the* number of *coins* of *each type.*

Sol. Let the number of 50 p, 25 P and 10 p coins be 5x, 9x and 4x respectively.
(5x/2)+(9x/ 4)+(4x/10)=206 ⇔ 50x + *45x* + 8x = 4120⇔1O3x = 4120 ⇔x=40.

Number of 50 p coins = (5 x 40) = 200; Number of 25 p coins = (9 x 40) = 360;
Number of 10 p coins = (4 x 40) = 160.

Ex. 7. *A* mixture *contains alcohol* and *water* in *the ratio* 4 : 3. *If* 5 litres of water *is* added *to the mixture, the ratio becomes* 4: 5. *Find the quantity of alcohol* in the given mixture

Sol. Let the quantity of alcohol and water be 4x litres and 3x litres respectively

4x/(3x+5)=4/5 ⇔20x=4(3x+5)⇔8x=20 ⇔x=2.5
Quantity of alcohol = (4 x 2.5) litres = 10 litres.

13. PARTNERSHIP

!IMPORTANT FACTS AND FORMULAE I

1. Partnership: When two or more than two persons run a business jointly, they are called partners and the deal is known as *partnership.*

2. Ratio of Division of Gains:

i) When investments of all the partners are for the same *time,* the gain or loss is distributed a among the partners in the ratio of their investments.
Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year:
(A's share of profit) : (B's share of profit) = x : y.

ii) ***When investments are for different time periods,*** then equivalent capitals are calculated for a unit of time by taking (capital x number of units of time). Now, gain or loss is divided in the ratio of these capitals.

Suppose A invests Rs. x for p months and B invests Rs. y for q months, then
(A's share of profit) : (B's share of profit) = xp : *yq.*

3. Working and Sleeping Partners: A partner who manages the business is known . as a *working partner* and the one who simply invests the money is a *sleeping partner.*

SOLVED EXAMPLES

Ex. 1. A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and ,Rs.1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.

Sol. Ratio of shares of A, Band C = Ratio of their investments
= 120000 : 135000 : 150000 = 8 : 9 : 10.

A's share = Rs. (56700 x (8/27))= Rs. 16800.

B's share = Rs. (56700 x (9/27)) = Rs. 18900.

C's share = Rs. (56700 x (10/27))=Rs. 21000.

Ex. 2. Alfred started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find the lire of each.

Sol. Clearly, Alfred invested his capital for 12 months, Peter for 9 months and Ronald for 3 months.

So, ratio of their capitals = (45000 x 12) : (60000 x 9) : (90000 x 3)
= 540000 : 540000 : 270000 = 2 : 2 : 1.

Alfred's share = Rs. (16500 x (2/5)) = Rs. 6600

Peter's share = Rs. (16500 x (2/5)) = Rs. 6600

Ronald's share = Rs. (16500 x (1/5)) = Rs. 3300.

Ex. 3. A, Band C start a business each investing Rs. 20,000. After 5 months A withdrew Rs.6000 B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. Find the share of each.

Sol. Ratio of the capitals of A, Band C
= 20000 x 5 + 15000 x 7 : 20000 x 5 + 16000 x 7 : 20000 x 5 + 26000 x 7
= 205000:212000 : 282000 = 205 : 212 : 282.

A's share = Rs. 69900 x (205/699) = Rs. 20500 I

B's share = Rs. 69900 x (212/699) = Rs. 21200;

C's share = Rs. 69900 x (282/699) = Rs. 28200.

Ex. 4. A, Band C enter into partnership. A invests 3 times as much as B and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B ?

Sol. Let C's capital = Rs. x. Then, B's capital = Rs. (2/3)x

A's capital = Rs. (3 x (2/3).*x*) = Rs. *2x.*

Ratio of their capitals = *2x* : (2/3)x :x = 6 : 2 : 3.

Hence, B's share = Rs. (6600 x (2/11))= Rs. 1200.

Ex. 5. Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Rs. 720, find the total rent of the field.

Sol. Ratio of shares of A, B, C, D = (24 x 3) : (10 x 5) : (35 x 4) : (21 x 3) = 72 : 50 : 140 : 63.

Let total rent be Rs. x. Then, *A's* share = Rs. (72x)/325

(72x)/325=720 ⇔ x=(720 x 325)/72 = 3250

Hence, total rent of the field is Rs. 3250.

Ex.6. A invested Rs. 76,000 in a business. After few months, B joined him Rs. 57,000. At the end of the year, the total profit was divided between them in ratio 2 : 1. After bow many months did B join?

Sol. Suppose B joined after x months. Then, B's money was invested for (12 - x)

(76000 x 12)/(57000 x (12-x) =2/1 ⇔ 912000=114000(12-x)

114 (12 - x) = 912⇔12-x=8⇔x=4

Hence, B joined after 4 months.

Ex.7. A, Band C enter into a partnership by investing in the ratio of 3 : 2: 4. After 1 year, B invests another Rs. 2,70,000 and C, at the end of 2 years, also invests Rs.2,70,000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find initial investment of each.

Sol. Let the initial investments of A, Band C be Rs. *3x,* Rs. 2x and Rs. 4x respectively. Then,

(3x x 36) : *[(2x* x 12) + *(2x* + 270000) x 24] : *[(4x* x 24) + *(4x* +270000) x 12]=3:4:5

1O8x : *(72x* + 6480000) : *(144x* + 3240000) = 3 : 4 : 5

108x /(72x+6480000)=3/4 ⇔ *432x* = *216x* + 19440000

⇔*216x* = 19440000

x=90000

Hence, A's initial investment = 3x = Rs. 2,70,000;

B's initial investment = 2x = Rs. 1,80,000;

C's initial investment = 4x = Rs. 3,60,000.

14. CHAIN RULE

_IMPORTANT FACTS AND FORMULAE

1. Direct Proportion: Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same

Ex. 1. Cost is directly proportional to the number of articles.

(More Articles, More Cost)

Ex. 2. Work done is directly proportional to the number of men working on it

(More Men, More Work)

2. Indirect Proportion: Two quantities are said to be indirectly proportional,if on the increase of the one, the other decreases to the same extent and vice-versa.

Ex. 1. The time taken by a car in covering a certain distance is inversely proportional to the speed of the car.

(More speed, Less is the time taken to cover a distance)

Ex. 2. Time taken to finish a work is inversely proportional to the num of persons working at it.

(More persons, Less is the time taken to finish a job)

Remark: In solving questions by chain rule, we compare every item with the term to be found out.

SOL VED EXAMPLES

Ex. 1. If 15 toys cost Rs, 234, what do 35 toys cost?

Sol. Let the required cost be Rs. x. Then,

More toys, More cost (Direct Proportion)

. 15 : 35 : : 234 : x ⇔ (15 x x) = (35 x 234) ⇔ x=(35 X 234)/15 =546

Hence, the cost of 35 toys is Rs. 546.

Ex. 2. If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it ?

Sol. Let the required number of hours be x. Then,

Less men, More hours (Indirect Proportion)

15 : 36 : : 25 : x ⇔(15 x x) = (36 x 25) ⇔(36 x 25)/15 = 60

Hence, 15 men can do it in 60 hours.

Ex. 3. If the wages of 6 men for 15 days be Rs.2100, then find the wages of for 12 days.

Sol. Let the required wages be Rs. x.

More men, More wages (Direct Proportion)
Less days, Less wages (Direct Proportion)

Men 6: 9 : :2100:x
Days 15:12

Therefore (6 x 15 x x)=(9 x 12 x 2100) ⇔ x=(9 x 12 x 2100)/(6 x 15)=2520

Hence the required wages are Rs. 2520.

Ex. 4. If 20 men can build a wall 66 metres long in 6 days, what length of a similar can be built by 86 men in 8 days?

Sol. Let the required length be x metres

More men, More length built (Direct Proportion)

Less days, Less length built (Direct Proportion)

Men 20: 35
Days 6: 3 : : 56 : x

Therefore (20 x 6 x x)=(35 x 3 x 56)⇔x=(35 x 3 x 56)/120=49

Hence, the required length is 49 m.

Ex. 5. If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

Sol. Let the required number of days be x.
More men, Less days (indirect proportion)
Less hours per day, More days (indirect proportion)

Men 18 : 15
Hours per day 8: 9 } : :16 : x
(18 x 8 x x)=(15 x 9 x 16)⇔ x=(44 x 15)144 = 15

Hence, required number of days = 15.

Ex. 6. If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours day, bow much coal will be required for 8 engines, each running 13hours a day, it being given that 3 engines of former type consume as much as 4 engines of latter type?

Sol. Let 3 engines of former type consume 1 unit in 1 hour.
Then, 4 engines of latter type consume 1 unit in 1 hour.

Therefore 1 engine of former type consumes(1/3) unit in 1 hour.

1 engine of latter type consumes(1/4) unit in 1 hour.

Let the required consumption of coal be x units.
Less engines, Less coal consumed (direct proportion)
More working hours, More coal consumed (direct proportion)
Less rate of consumption, Less coal consumed(direct prportion)

Number of engines 9: 8
Working hours 8 : 13 } :: 24 : x
Rate of consumption (1/3):(1/4)

[9 x 8 x (1/3) x x) = (8 x 13 x (1/4) x 24) ⇔ 24x = 624 ⇔ x = 26.

Hence, the required consumption of coal = 26 metric tonnes.

Ex. 7. A contract is to be completsd in 46 days sad 117 men were said to work 8 hours a day. After 33 days, (4/7) of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day?

Sol. Remaining work = (1-(4/7) =(3/7)

Remaining period = (46 - 33) days = 13days

Let the total men working at it be x.

Less work, Less men (Direct Proportion)
Less days, More men (Indirect Proportion)
More Hours per Day, Less men (Indirect Proportion)

Work (4/7): (3/7)
Days 13:33 } : : 117: x
Hrs/day 9 : 8

Therefore (4/7) x 13 x 9 x x =(3/7) x 33 x 8 x 117 or x=(3 x 33 x 8 x 117)/(4 x 13 x 9)=198

Additional men to be employed = (198 - 117) = 81.

Ex. 8. A garrison of 3300 men had provisions for 32 days, when given at the rate of 860 gns per head. At the end of 7 days, a reinforcement arrives and it was for that the provisions wi1l last 17 days more, when given at the rate of 826 gms per head, What is the strength of the reinforcement?

Sol. The problem becomes:
3300 men taking 850 gms per head have provisions for (32 - 7) or 25 days,
How many men taking 825 gms each have provisions for 17 days?

Less ration per head, more men (Indirect Proportion)
Less days, More men (Indirect Proportion)

Ration 825 : 850

Days 17: 25 } : : 3300 : x

(825 x 17 x x) = 850 x 25 x 3300 or x = (850 x 25 x 3300)/(825 x 17)=5000

Strength of reinforcement = (5500 - 3300) = 1700.

15. TIME AND WORK

IIMPORTANT FACTS AND FORMULAE

1. If A can do a piece of work in n days, then A's 1 day's work = (1/n).

2. If A's 1 day's work = (1/n),then A can finish the work in n days.

3. A is thrice as good a workman as B, then:
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED EXAMPLES

Ex. 1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same Job.How long should it take both A and B, working together but independently, to do the same job? (IGNOU, 2003)

Sol. A's 1 hour's work = 1/8

B's 1 hour's work = 1/10

(A + B)'s 1 hour's work = (1/8) +(1/10)=9/40

Both A and B will finish the work in 40/9 days.

Ex. 2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work? (Bank P.O. 2003)

Sol. (A + B)'s 1 day's work = (1/4). A's 1 day's work = (1/12).

B's 1 day's work =((1/4)-(1/12))=(1/6)

Hence, B alone can complete the work in 6 days.

Ex. 3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days

of 7 bours each. How long will they take to do it, working together 8 hours a day?

Sol. A can complete the work in (7 x 9) = 63 hours.
B can complete the work in (6 x 7) = 42 hours.

A's 1 hour's work = (1/63) and B's 1 hour's work =(1/42)

(A + B)'s 1 hour's work =(1/63)+(1/42)=(5/126)

Both will finish the work in (126/5) hrs.
Number of days. of (42/5) hrs each =(126 x 5)/(5 x 42)=3 days

Ex. 4. A and B can do a piece of work in 18 days; Band C can do it in 24 days A and C can do it in 36 days. In how many days will A, Band C finish it together and separately?

Sol. (A + B)'s 1 day's work = (1/18)   (B + C)'s 1 day's work = (1/24)
and (A + C)'s 1 day's work = (1/36)

Adding, we get: 2 (A + B + C)'s 1 day's work =(1/18 + 1/24 + 1/36)

=9/72 =1/8

(A +B + C)'s 1 day's work =1/16

Thus, A, Band C together can finish the work in 16 days.
Now, A's 1 day's work = [(A + B + C)'s 1 day's work] - [(B + C)'s 1 day work:
=(1/16 – 1/24)= 1/48

A alone can finish the work in 48 days.

Similarly, B's 1 day's work =(1/16 – 1/36)=5/144

B alone can finish the work in 144/5=28 4/5 days

And C's 1 day work =(1/16-1/18)=1/144

Hence C alone can finish the work in 144 days.

Ex. 6. A is twice as good a workman as B and together they finish a piece in 18 days. In how many days will A alone finish the work?
Sol. (A's 1 day's work):)(B's 1 days work) = 2 : 1.

(A + B)'s 1 day's work = 1/18

Divide 1/18 in the ratio 2 : 1.

:. A’s 1 day's work =(1/18*2/3)=1/27

Hence, A alone can finish the work in 27 days.

Ex. 6. A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

Sol. Ratio of times taken by A and B = 160 : 100 = 8 : 5.

Suppose B alone takes x days to do the job.

Then, 8 : 5 :: 12 : x = 8x = 5 x 12 =x = 7 1/2 days.

Ex. 7. A can do a piece of work in 80 days. He works at it for 10 days B alone finishes the remaining work in 42 days. In how much time will A and B working together, finish the work?

Sol. Work done by A in 10 days =(1/80*10)=1/8

Remaining work = (1- 1/8) =7/ 8

Now,7/ 8 work is done by B in 42 days.

Whole work will be done by B in (42 x 8/7) = 48 days.
A’s 1 day's work = 1/80 and B's 1 day's work = 1/48

(A+B)'s 1 day's work = (1/80+1/48)=8/240=1/30
Hence, both will finish the work in 30 days.

Ex. 8. A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. !find the share of each.

Sol :C's 1 day's work = 1/3-(1/6+1/8)=24

A : B : C = Ratio of their 1 day's work = 1/6:1/8:1/24= 4 : 3 : 1.
A’s share = Rs. (600 *4/8) = Rs.300, B's share = Rs. (600 *3/8) = Rs. 225.

C's share = Rs. [600 - (300 + 225») = Rs. 75.

Ex. 9. A and B working separately can do a piece of work in 9 and 12 days respectively, If they work for a day alternately, A beginning, in how many days, the work will be completed?

(A + B)'s 2 days' work =(1/9+1/12)=7/36

Work done in 5 pairs of days =(5*7/36)=35/36

Remaining work =(1-35/36)=1/36

On 11th day, it is A’s turn. 1/9 work is done by him in 1 day.

1/36 work is done by him in(9*1/36)=1/4 day

Total time taken = (10 + 1/4) days = 10 1/4days.

Ex 10 .45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

(45 x 16) men can complete the work in 1 day.

1 man's 1 day's work = 1/720

45 men's 6 days' work =(1/16*6)=3/8

Remaining work =(1-3/8)=5/8

75 men's 1 day's work = 75/720=5/48

Now, $\frac{5}{48}$ work is done by them in 1 day.

$\frac{5}{8}$ work is done by them in $\left(\frac{48}{5} \times \frac{5}{8}\right)$=6 days.

Ex:11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days.In how many days can 2 men and 1 boy do the work?

Soln: Let 1 man’s 1 day’s work = x and 1 boy’s 1 day’s work = y.

Then, $2x+3y = \frac{1}{10}$ and $3x+2y = \frac{1}{8}$

Solving,we get: $x = \frac{7}{200}$ and $y = \frac{1}{100}$

(2 men + 1 boy)’s 1 day’s work $= \left(2 \times \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}$

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days

16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. **Inlet:** A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.
Outlet: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (i) If a pipe can fill a tank in x hours, then : part filled in 1 hour = 1/x

(ii) If a pipe can empty a full tank in *y* hours, then : part emptied in 1 hour = 1/y

(iii) If a pipe can .fill a tank in x hours and another pipe can empty the full tank in y hours (where *y> x),* then on opening both the pipes, the net part filled in 1 hour = (1/x)-(1/y)

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where x > *y),* then on opening both the pipes, the net part emptied in 1 hour = (1/y)-(1/x)

SOLVED EXAMPLES

Ex. 1:Two pipes A and B can fill a tank in 36 bours and 46 bours respectively. If both the pipes are opened simultaneously, bow mucb time will be taken to fill the tank?

Sol: Part filled by A in 1 hour = (1/36);

Part filled by B in 1 hour = (1/45);

Part filled by (A + B) In 1 hour =(1/36)+(1/45)=(9/180)=(1/20)

Hence, both the pipes together will fill the tank in 20 hours.

Ex. 2: Two pipes can fill a tank in 10hours and 12 hours respectively while a third, pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?

Sol: Net part filled In 1 hour =(1/10)+(1/12)-(1/20)=(8/60)=(2/15).
The tank will be full in 15/2 hrs = 7 hrs 30 min.

Ex. 3: If two pipes function simultaneously, tbe reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than tbe otber. How many hours does it take the second pipe to fill the reservoir?

Sol:let the reservoir be filled by first pipe in x hours.

Then ,second pipe fill it in (x+10)hrs.

Therefore (1/x)+(1/x+10)=(1/12) ⇔(x+10+x)/(x(x+10))=(1/12).

⇔ x^2 –14x-120=0 ⇔ (x-20)(x+6)=0

⇔x=20 [neglecting the negative value of x]

so, the second pipe will take (20+10)hrs. (i.e) 30 hours to fill the reservoir

Ex. 4: A cistern has two taps which fill it in 12 minutes and 15minutes respectively. There is also a waste pipe in the cistern. When all the 3 are opened , the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol: Workdone by the waste pipe in 1min

=(1/20)-(1/12)+(1/15) = -1/10 [negative sign means emptying]

therefore the waste pipe will empty the full cistern in 10min

Ex. 5: An electric pump can fill a tank in 3 hours. Because of a leak in ,the tank it took 3(1/2) hours to fill the tank. If the tank is full, how much time will the leak take to empty it ?

Sol: work done by the leak in 1 hour=(1/3)-(1/(7/2))=(1/3)-(2/7)=(1/21).

The leak will empty .the tank in 21 hours.

Ex. 6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it tooki 32 minutes more to fill the cistern.When the cistern is full, in what time will the leak empty it?

Sol: Work done by the two pipes in 1 hour =(1/14)+(1/16)=(15/112).

Time taken by these pipes to fill the tank = (112/15) hrs = 7 hrs 28 min.

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

Work done by (two pipes + leak) in 1 hour = (1/8).

Work done by the leak m 1 hour =(15/112)-(1/8)=(1/112).

Leak will empty the full cistern in 112 hours.

Ex. 7: Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. after 7 min,C is also opened. In how much time, the tank is full?

Sol: Part filled in 7 min. = 7*((1/36)+(1/45))=(7/20).

Remaining part=(1-(7/20))=(13/20).

Net part filled in 1min. when A,B and C are opened=(1/36)+(1/45)-(1/30)=(1/60).

Now,(1/60) part is filled in one minute.

(13/20) part is filled in (60*(13/20))=39 minutes.

Ex.8: Two pipes A,B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 min.?

Sol: let B be closed after x min. then ,

Part filled by (A+B) in x min. +part filled by A in (18-x)min.=1

Therefore x*((1/24)+(1/32))+(18-x)*(1/24)=1 ⇔ (7x/96) + ((18-x)/24)=1.

⇔ 7x +4*(18-x)=96.

Hence, be must be closed after 8 min.

17. TIME AND DISTANCE

IMPORTANT FACTS AND FORMULAE

1. Speed = $\left(\frac{\text{Distance}}{\text{Time}}\right)$, Time= $\left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance = (Speed * Time)

2. x km / hr = $x * \frac{5}{18}$

3. x m/sec = (x * 18/5) km /hr

4. If the ratio of the speeds of A and B is a:b , then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or b:a.

5. Suppose a man covers a certain distance at x km/ hr and an equal distance at y km / hr . Then , the average speed during the whole journey is $\frac{2xy}{x+y}$ km/ hr.

SOLVED EXAMPLES

Ex. 1. How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?

Sol. Aditya's speed = 20 km/hr = $\{20 * \frac{5}{18}\}$ m/sec = $\frac{50}{9}$ m/sec

$\therefore$ Time taken to cover 400 m= $\{ 400 * \frac{9}{50} \}$ sec =72 sec = $1\frac{12}{60}$ min $1\frac{1}{5}$ min.

Ex. 2. A cyclist covers a distnce of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

Sol. Speed = $\{ \frac{750}{150} \}$ m/sec =5 m/sec = $\{ 5 * \frac{18}{5} \}$ km/hr =18km/hr

Ex. 3. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Sol. Let the distance covered in 1 leap of the dog be x and that covered in 1 leap of the hare by y.

Then , 3x = 4y => x = $\frac{4}{3}$ y => 4x = $\frac{16}{3}$ y.

$\therefore$ Ratio of speeds of dog and hare = Ratio of distances covered by them in the same time

= 4x : 5y = $\frac{16}{3}$ y : 5y = $\frac{16}{3}$: 5 = 16:15

Ex. 4.While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his speed in metres per second?

Sol. Let the speed be x km/hr.

Then, distance covered in 1 hr. 40 min. i.e., $1\frac{2}{3}$ hrs $= \frac{5x}{3}$ km

Remaining distance $= \{24 - \frac{5x}{3}\}$ km.

$$\therefore \frac{5x}{3} = \frac{5}{7}\{24 - \frac{5x}{3}\} \Leftrightarrow \frac{5x}{3} = \frac{5}{7}\{\frac{72-5x}{3}\} \Leftrightarrow 7x = 72 - 5x$$

$$\Leftrightarrow 12x = 72 \Leftrightarrow x=6$$

Hence speed = 6 km/hr $=\{6 * \frac{5}{18}\}$ m/sec $= \frac{5}{3}$ m/sec $= 1\frac{2}{3}$

Ex. 5.Peter can cover a certain distance in 1 hr. 24 min. by covering two-third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance.

Sol. Let the total distance be x km . Then,

$$\frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5} \Leftrightarrow \frac{x}{6} + \frac{x}{15} = \frac{7}{5} \Leftrightarrow 7x = 42 \Leftrightarrow x = 6$$

Ex. 6.A man traveled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.

Sol. Average speed $= \{\frac{2xy}{x+y}\}$ km/hr $=\{\frac{2*25*4}{25+4}\}$ km/hr $= \frac{200}{29}$ km/hr

Distance traveled in 5 hours 48 minutes i.e., $5\frac{4}{5}$ hrs. $= \{\frac{200}{29} * \frac{29}{5}\}$ km $= 40$ km

Distance of the post-office from the village $=\{\frac{40}{2}\} = 20$ km

Ex. 7.An aeroplane files along the four sides of a square at the speeds of 200,400,600 and 800km/hr.Find the average speed of the plane around the field.

Sol. :

Let each side of the square be x km and let the average speed of the plane around the field by y km per hour then ,

$x/200+x/400+x/600+x/800=4x/y \Leftrightarrow 25x/2500 \Leftrightarrow 4x/y \Leftrightarrow y=(2400*4/25)=384$

hence average speed =384 km/hr

Ex. 8.Walking at $\frac{5}{7}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Sol. : New speed =5/6 of the usual speed

New time taken=6/5 of the usual time

So,(6/5 of the usual time)-(usual time)=10 minutes.
=>1/5 of the usual time=10 minutes.

- ⇨ usual time=10 minutes

Ex. 9.If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However, if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.
Sol. Let the required distance be x km
Difference in the time taken at two speeds=1 min =1/2 hr
Hence x/5-x/6=1/5<=>6x-5x=6
⇔x=6
Hence, the required distance is 6 km

Ex. 10. A and B are two stations 390 km apart. A train starts from A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?
Sol. Suppose they meet x hours after 10 a.m. Then,
(Distance moved by first in x hrs) + [Distance moved by second in (x-1) hrs]=390.

$65x + 35(x-1) = 390 \Rightarrow 100x = 425 \Rightarrow x = 17/4$

So, they meet 4 hrs.15 min. after 10 a.m i.e., at 2.15 p.m.

Ex. 11. A goods train leaves a station at a certain time and at a fixed speed. After ^hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.
Sol. Let the speed of the goods train be x kmph.
Distance covered by goods train in 10 hours= Distance covered by express train in 4 hours
10x = 4 x 90 or x =36.
So, speed of goods train = 36kmph.

Ex. 12. A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8km/hr and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?
Sol. Relative speed of the policeman = (10-8) km/hr =2 km/hr.
Time taken by police man to cover 100m $\left(\frac{100}{1000} \times \frac{1}{2}\right)$ hr $= \frac{1}{20}$ hr.
In $\frac{1}{20}$ hrs, the thief covers a distance of $8 \times \frac{1}{20}$ km $= \frac{2}{5}$ km = 400 m

Ex.13. I walk a certain distance and ride back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long would it take me to ride both ways?

Sol. Let the distance be x km. Then,

(Time taken to walk x km) + (time taken to ride x km) =37 min.

(Time taken to walk 2x km) + (time taken to ride 2x km)= 74 min.

But, the time taken to walk 2x km = 55 min.

Time taken to ride 2x km = (74-55)min =19 min.

18. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE

1. *a km/hr*= *(a** 5/18) m/s.

2. a m */ s* = (a*18/5) km/hr.

3 Time taken by a train of length *1* metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover *1* metres.

4. Time taken by a train of length *1* metres to pass a stationary object of length *b* metres is the time taken by the train to cover *(1 + b)* metres.

5. Suppose two trains or two bodies are moving in the same direction at u m / s and *v m/s,* where u > *v,* then their relatives speed = (u - *v)* m / s.

6. Suppose two trains or two bodies are moving in opposite directions at u m / s and *v m/s,* then their relative speed is = (u + *v)* m/s.

7. If two trains of length *a* metres and *b* metres are moving in opposite directions at u m */* s and *v* m/s, then time taken by the trains to cross each other = *(a + b)/(u+v)* sec.

8.If two trains of length a metres and *b* metres are moving in the same direction at u m / s and *v* m / s, then the time taken by the faster train to cross the slower train = (a+b)/(u-v) sec.

9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take *a* and *b* sec in reaching B and A respectively, then

(A's speet) : *(B's speed)* = $(b^{1/2} : a^{1/2})$.

SOLVED EXAMPLES

Ex.I. A train 100 m long is running at the speed of 30 km / hr. Find the time taken by it to pass a man standing near the railway line. (S.S.C. 2001)

Sol. Speed of the train = (30 x 5/18) m / sec = (25/3) m/ sec.

Distance moved in passing the standing man = 100 m.

Required time taken = *100/(25/3)* = (100 *(3/25)) sec = 12 sec

Ex. 2. A train is moving at a speed of 132 km/br. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long? (Section Officers', 2003)

Sol. Speed of train = 132 *(5/18) m/sec = 110/3 m/sec.

Distance covered in passing the platform = (110 + 165) m = 275 m.

Time taken =275 *(3/110) sec =15/2 sec = 7 ½ sec

Ex. 3. A man is standing on a railway bridge which is 180 m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed?

Sol. Let the length of the train be x metres,

Then, the train covers x metres in 8 seconds and (x + 180) metres in 20 sec

x/8=(x+180)/20 ⇔ 20x = 8 *(x* + 180) <=> *x* = 120.

Length of the train = 120 m.

Speed of the train = (120/8) m / sec = m / sec = (15 *18/5) kmph = 54 km

Ex. 4. A train 150 m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Sol: Speed of the train relative to man = (68 - 8) kmph

= (60* 5/18) m/sec = (50/3)m/sec

Time taken by the train to cross the man I

= Time taken by It to cover 150 m at 50/3 m / sec = 150 *3/ 50 sec = 9sec

Ex. 5. A train 220 m long is running with a speed of 59 kmph.. In what will

it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

sol. Speed of the train relative to man = (59 + 7) kmph

= 66 *5/18 *m/sec* = 55/3 m/sec.

Time taken by the train to cross the man = Time taken by it to cover 220 m at (55/3) m / sec = (220 *3/55) sec = 12 sec

Ex. 6. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmpb. In what time will they be clear of each other from the moment they meet?

Sol. Relative speed of the trains = (42 + 48) kmph = 90 kmph

=(90*5/18) m / sec = 25 m /sec.

Time taken by the trains to'pass each other

= Time taken to *cover* (137 + 163) m at 25 m */sec* =(300/25) sec = 12 sec

Ex. 7. Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr,In howmuch time will the first train cross the second?

Sol: Relative speed of the trains = (72 - 54) km/hr = 18 *km/hr*

= (18 * 5/18) *m/sec* = 5 *m/sec.*

Time taken by the trains to cross each other

= Time taken to cover (100 + 120) m at 5 m /sec = (220/5) sec = 44 sec.

Ex. 8. A train 100 metres long takes 6 seconds to cross a man walking at 5 kmph in the direction opposite to that of the train. Find the speed of the

train.?

Sol:Let the speed of the train be x kmph.

Speed of the train relative to man = (x + 5) kmph = $(x + 5)$ *5/18 m/sec.

Therefore 100/((x+5)*5/18)=6 <=> 30 $(x + 5)$ = 1800 <=> x = 55

Speed of the train is 55 kmph.

Ex9. A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes.12 sec to pass a man walking at 6 kmph in the same direction in which the train is going . Find the length of the train and the length of the platform.

Sol: Let the length of train be x metres and length of platform be y metres.

Speed of the train relative to man = (54 - 6) kmph = 48 kmph

= 48*(5/18) m/sec = 40/3 m/sec.

In passing a man, the train covers its own length with relative speed.

Length of train = (Relative speed * Time) = (40/3)*12 m = 160 m.

Also, speed of the train = 54 *(5/18)m / sec = 15 m / sec.

(x+y)/15 = 20 <=> $x + y$ = 300 <=> Y = (300 - 160) m = 140 m.

Ex10. A man sitting in a train which is traveling at 50 kmph observes that a goods train, traveling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.?

Sol: Relative speed = 280/9 m / sec = ((280/9)*(18/5)) kmph = 112 kmph.

Speed of goods train = (112 - 50) kmph = 62 kmph.

19.BOATS AND STREAMS

IMPORTANT FACTS AND FORMULAE

1.In water ,the direction along the stream is called downstream and ,the direction against the stream is called upstream.

2.If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr,then:

speed downstream=(u+v)km/hr.
speed upstream=(u-v)km/hr.

3.If the speed downstream is a km/hr and the speed upstream is b km/hr,then :
speed in still water=1/2(a+b)km/hr
rate of stream=1/2(a-b)km/hr

SOLVED EXAMPLES

EX.1.A man can row upstream at 7 kmph and downstream at 10kmph.find man's rate in still water and the rate of current.

Sol. Rate in still water=1/2(10+7)km/hr=8.5 km/hr.
Rate of current=1/2(10-7)km/hr=1.5 km/hr.

EX.2. A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2hours30minutes to cover a distance of 5km upstream. find the speed of the river current in km/hr.

Sol. rate downstream=(15/3 ¾)km/hr=(15*4/15)km/hr=4km/hr.
Rate upstream=(5/2 ½)km/hr=(5*2/5)km/hr=2km/hr.
Speed of current=1/2(4-2)km/hr=1km/hr

EX.3. a man can row 18 kmph in still water.it takes him thrice as long to row up as to row down the river.find the rate of stream.

Sol. Let man's rate upstream be x kmph.then ,his rate downstream=3xkmph.
So,2x=18 or x=9.
Rate upstream=9 km/hr,rate downstream=27 km/hr.
Hence,rate of stream=1/2(27-9)km/hr=9 km/hr.

EX.4. there is a road beside a river.two friends started from a place A,moved to a temple situated at another place B and then returned to A again.one of them moves

on a cycle at a speed of 12 km/hr,while the other sails on a boat at a speed of 10 km/hr.if the river flows at the speed of 4 km/hr,which of the two friends will return to placeA first?

Sol. Clearly the cyclist moves both ways at a speed of 12 km/hr.

The boat sailor moves downstream @ (10+4)i.e.,14 km/hr and upstream @ (10-4)i.e., 6km/hr.

So,average speed of the boat sailor=(2*14*6/14+6)km/hr

=42/5 km/hr=8.4 km/hr.

since the average speed of the cyclist is greater ,he will return ta A first.

EX.5. A man can row 7 ½ kmph in still water.if in a river running at 1.5 km/hr an hour,it takes him 50 minutes to row to a place and back,how far off is the place?

Sol. Speed downstream =(7.5+1.5)km/hr=9 km/hr;

Speed upstream=(7.5-1.5)kmph=6kmph.

Let the required distance be x km.then,

x/9+x/6=50/60.

2x+3x=(5/6*18)

5x=15

x=3.

Hence,the required distance is 3km.

EX.6. In a stream running at 2kmph,a motar boat goes 6km upstream and back again to the starting point in 33 minutes.find the speed of the motarboat in still water.

Sol.let the speed of the motarboat in still water be x kmph.then,

6/x+2 +6/x-2=33/60

$11x^2$-240x-44=0

$11x^2$-242x+2x-44=0

(x-22)(11x+2)=0

x=22.

EX.7.A man can row 40km upstream and 55km downstream in 13 hours also, he can row 30km upstream and 44km downstream in 10 hours.find the speed of the man in still water and the speed of the current.

Sol.let rate upstream=x km/hr and rate downstream=y km/hr.

Then,40/x +55/y =13…(i) and 30/x +44/y =10

Multiplying (ii) by 4 and (i) by 3 and subtracting ,we get:11/y=1 or y=11.

Substituting y=11 in (i),we get:x=5.

Rate in still water =1/2(11+5)kmph=8kmph.

Rate of current=1/2(11-5)kmph=3kmph

20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULAE

1. **Alligation**: It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.
2. **Mean Price**: The cost price of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation**: If two ingredients are mixed, then

$$\frac{\text{(Quantity of cheaper)}}{\text{(Quantity of dearer)}} = \frac{\text{(C.P. of dearer) - (Mean price)}}{\text{(Mean price) - (C.P. of cheaper)}}$$

We present as under:

C.P. of a unit quantity of cheaper　　　　C.P. of a unit quantity of dearer

(c)　　　　　　　　　　　　　　　　(d)

(m)

(d-m)　　　　　　　　　　　　　　　　*(m-c)*

$\therefore$ (Cheaper quantity) : (Dearer quantity) = (d - m) : (m - *c*).

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations the quantity of pure liquid= $\left[\ x(1\text{-}y/x)\text{^}n\right]$ units.

SOLVED EXAMPLES

Ex. 1. In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg ?

Sol. By the rule of alligation, we have:

C.P. of 1 kg rice of 1st kind (in paise)　　　　C.P. of 1 kg rice of 2nd kind (in paise)

930 1080 .

.

Mean pnce
(in paise)
1000

80 70

:.' Required ratio = 80 : 70 = 8 : 7.

Ex. 2. How much water must be added to 60 litres of milk at 1 ½ litres for Rs. 2 So as to have a mixture worth Rs.10 2/3 a litre ?

Sol. C.P. of 1 litre of milk = Rs. (20 x 2/3) = Rs. 40/3

c.p of 1 litre of milk c.p of 1 litre of milk

0 Rs.$\frac{40}{3}$

Mean price
(Rs. $\frac{32}{3}$)

(40/3-32/3)=8/3 (32/3-0)=32/3

∴ Ratio of water and milk = $\frac{8}{3}$: $\frac{32}{3}$ = 8 : 32 = 1 : 4

∴ Quantity of water to be added to 60 litres of milk = $\left[1/4 \text{ X } 60\right]$ litres =15 litre

Ex. 3. In what ratio must water be mixed with milk to gain 20 % by selling the mixture at cost price?

Sol. Let C.P. of milk be Re. 1 per litre.
Then, S.P. of 1 litre of mixture = Re. 1.
Gain obtained = 20%.

∴ C.P. of 1 litre of mixture = Rs.$\left[(100/120)^* \ 1\right]$=Rs.5/6

By the rule of alligation, we have:

C.P. of 1 litre of water   C.P. of1litre ofmilk

0   Re.1

(Re. 5/6)

(1- (5/6))= 1/6   ((5/6)-0)=5/6

∴Ratio of water and milk = 1/6 : 5/6 =

Ex. 4. .How many kgs. of wheat costing Rs. 8 per kg must be mixed with 86 kg of rice costing Rs. 6.40 per kg so that 20% gain may be obtained by Belling the mixture at Rs. 7.20 per kg ?

Sol. S.P. of 1 kg mixture = Rs. 7.20,Gain = 20%.

∴ C.P. of 1 kg mixture = Rs.$\left[(100/120)*7.20\right]$=Rs. 6.

By the rule of alligation, we have:

C_P. of 1 kg wheat of 1st kind   C.P. of 1 kg wheat of 2nd kind

(800p) .   (540 p)

Mean price

(600 p)

60   200

Wheat of 1st kind: Wheat of 2nd kind = 60 : 200 = 3 : 10.

Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind.

Then, 3 : 10 = x : 36 or lOx = 3 * 36 or x = 10.8 kg.

Ex. 5. The milk and water in two vessels A and B are in the ratio 4 : 3 and 2: 3 respectively. In

what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water?

Sol. Let the C.P. of milk be Re. 1 per litre

Milk in 1 litre mixture of A = 4/7litre; Milk in 1 litre mixture of B = 2/5 litre;
Milk in 1 litre mixture of C = ½ litre
C.P. of 1 litre mixture in A = Re .4/7*;* C.P. of 1 litre mixture in B = Re.2/5
Mean price = Re.1/2
By the rule of alligation, we have:

C.P. of 1 litre mix. in A   C.P. of 1 litre mix. in B

(4/7)   (2/5)

(1/2)

(1/10)   (1/14)

Required ratio = 1/10 : 1/14 = 7 : 5

21. SIMPLE INTEREST

IMPORTANT FACTS AND FORMULAE

1.. **Principal**: The money borrowed or lent out for a certain period is called the **principal** or the **sum.**
2**. Interest**: Extra money paid for using other's money is called **interest.**
3. **Simple Interest (S.I.)** : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called **simple interest.**
Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then,

(i) S.I. = (P*R*T)/100
(ii) P=(100*S.I)/(R*T) ;R=(100*S.I)/(P*T) and T=(100*S.I)/(P*R)

SOLVED EXAMPLES

Ex. 1. **Find the simple interest on Rs. 68,000 at 16 2/3% per annum for 9 months.**

Sol. P = Rs.68000,R = 50/3% p.a and T = 9/12 years = 3/4years.

$\therefore$ S.I. = (P*R*T)/100 = Rs.(68,000*(50/3)*(3/4)*(1/100))= Rs.8500

Ex. 2. Find the simple interest on Rs. 3000 at 6 1/4% per annum for the period from 4th Feb., 2005 to 18th April, 2005.

Sol. Time = (24+31+18)days = 73 days = 73/365 years = 1/5 years.

P = Rs.3000 and R = 6 ¼ %p.a = 25/4%p.a

$\therefore$S.I. = Rs.(3,000*(25/4)*(1/5)*(1/100))= Rs.37.50.

Remark : The day on which money is deposited is not counted while the day on which money is withdrawn is counted .

Ex. 3. A sum at simple interests at 13 ½ % per annum amounts to Rs.2502.50 after 4 years find the sum.

Sol. Let sum be Rs. x then , S.I.=Rs.(x*(27/2) *4*(1/100)) = Rs.27x/50

$\therefore$ amount = $\left(\text{Rs. } x+(27x/50)\right)$ = Rs.77x/50

$\therefore$ 77x/50 = 2502.50 $\Leftrightarrow$ x = $\frac{2502.50 * 50}{77}$ = 1625

Hence , sum = Rs.1625.

Ex. 4. A sum of Rs. 800 amounts to Rs. 920 in 8 years at simple intere interest rate is increased by 8%, it would amount to bow mucb ?

Sol. S.l. = Rs. (920 - 800) = Rs. 120; p = Rs. 800, T = 3 yrs. _

. R = $\left((100 \text{ x } 120)/(800*3)\right)$ % = 5%.

New rate = (5 + 3)% = 8%.
New S.l. = Rs. (800*8*3)/100 = Rs. 192.

: New amount = Rs.(800+192) = Rs. 992.

Ex. 5. Adam borrowed some money at the rate of 6% p.a. for the first two years , at the rate of 9% p.a. for the next three years , and at the rate of 14% p.a. for the period beyond five years. 1£ he pays a total interest of Rs. 11, 400 at the end of nine years how much money did he borrow ?

Sol. Let the sum borrowed be x. Then,

(x*2*6)/100 + (x*9*3)/100 + (x*14*4)/100 = 11400

$\Leftrightarrow$ $\left(3x/25 + 27x/100 + 14x / 25\right)$ = 11400 $\Leftrightarrow$ 95x/100 = 11400 $\Leftrightarrow$ x = (11400*100)/95 = 12000.

Hence , sum borrowed = Rs.12,000.

Ex. 6. A certain sum of money amounts to Rs. 1008 in 2 years and to Rs.1164 in 3 ½ years. Find the sum and rate of interests.

Sol.. S.I. for 1 ½ years = Rs.(1164-1008) = Rs.156.

S.l. for 2 years = Rs.(156*(2/3)*2)=Rs.208

Principal = Rs. (1008 - 208) = Rs. 800.

Now, P = 800, T = 2 and S.l. = 208.

Rate =(100* 208)/(800*2)% = 13%

Ex. 7. At what rate percent per annum will a sum of money double in 16 years.

Sol.. Let principal = P. Then, S.l. = P and T = 16 yrs.

$\therefore$ Rate = (100 x P)/(P*16)% = 6 ¼ % p.a.

Ex. 8. The simple interest on a sum of money is 4/9 of the principal .Find the rate percent and time, if both are numerically equal.

Sol. Let sum = Rs. x. Then, S.l. = Rs. 4x/9

Let rate = R% and time = R years.

Then, (x*R*R)/100=4x/9 or R^2 =400/9 or R = 20/3 = 6 2/3.

$\therefore$ Rate = 6 2/3 % and Time = 6 2/3 years = 6 years 8 months.

Ex. 9. The simple interest on a certain sum of money for 2 l/2 years at 12% per annum is Rs. 40 less tban the simple interest on the same sum for 3 ½ years at 10% per annum. Find the sum.

Sol. Let the sum be Rs. x Then, $\left((x*10*7)/(100*2)\right) - \left((x*12*5)/(100*2)\right) = 40$

$\Leftrightarrow$ (7x/20)-(3x/10)=40
$\Leftrightarrow x$ = (40 * 20) = 800.

Hence, the sum is Rs. 800.

Ex. 10. A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate, it would have fetched Rs. 360 more. Find the sum.

Sol. Let sum = P and original rate = R.

Then, $\left[(P*(R+2)*3)/100\right] - \left[(P*R*3)/100\right] = 360.$

$\Leftrightarrow$ 3PR + 6P - 3PR = 36000 $\Leftrightarrow$ 6P=36000 $\Leftrightarrow$ P=6000

Hence, sum = Rs. 6000.

Ex. 11. What annual instalment will discharge a debt of Rs. 1092 due in 3 years at 12% simple interest?

.

Sol .Let each Instalment be Rs. x

Then, $\left(x+\left((x*12*1)/100\right)\right) + \left(x+\left((x*12*2)/100\right)\right) + x = 1092$

$\Leftrightarrow ((28x/25) + (31x/25) + x) = 1092 \quad \Leftrightarrow (28x+31x+25x)=(1092*25)$

$\Leftrightarrow$ x= (1092*25)/84 = Rs.325.

$\therefore$ Each instalment = Rs. 325.

Ex. 12. A sum of Rs. 1550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is Rs. 106, find the money lent at each rate.

Sol. Let the sum lent at 8% be Rs. x and that at 6% be Rs. (1550 - *x).*

$\therefore \big((x*8*1)/100\big) + \big((1550-x)*6*1\big)/100=106$

$\Leftrightarrow 8x + 9300 - 6x=10600 \Leftrightarrow 2x = 1300 \Leftrightarrow x = 650.$

$\therefore$ Money lent at 8% = Rs. 650. Money lent at 6% = Rs. (1550 - 650) = Rs. 900.

22.COMPOUND INTEREST

Compound Interest: Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit,the amount after second unit becomes the principal for the third unit and so on.

After a specified period, ***the difference between the amount and the money borrowed is called the*** **Compound** ***Interest (abbreviated*** **as** ***C.I.) for that period.***

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compound Annually:

Amount = $P(1+R/100)^n$

II. When interest is compounded Half-yearly:

Amount = $P[1+(R/2)/100]^{2n}$

III. When interest is compounded Quarterly:

Amount = $P[1+(R/4)/100]^{4n}$

IV. When interest is compounded AnnuaI1y but time is in fraction, say 3(2/5) years.

Amount = $P(1+R/100)^3 \times (1+(2R/5)/100)$

V. When Rates are different for different years, say Rl%, R2%, R3% for 1st, 2nd and 3rd year respectively.

Then, Amount = $P(1+R_1/100)(1+R_2/100)(1+R_3/100)$

VI. Present worth of Rs.x due *n* years hence is given by :

Present Worth = $x/(1+(R/100))^n$

SOLVED EXAMPLES

Ex.1. ***Find compound interest* on *Rs. 7500* at 4% *per* annum *for* 2 *years, compounded* annually.**

Sol.

Amount = Rs [7500*(1+(4/100)^2] = Rs (7500 * (26/25) * (26/25)) = Rs. 8112.

therefore, C.I. = Rs. (8112 - 7500) = Rs. 612.

Ex. 2. ***Find compound interest* on *Rs. 8000* at 15% *per* annum *for* 2 *years* 4 *months, compounded annually*.**

Sol. Time = 2 years 4 months = 2(4/12) years = 2(1/3) years.

Amount = Rs'. [8000 X (1+(15/100))^2 X (1+((1/3)*15)/100)]

=Rs. [8000 * (23/20) * (23/20) * (21/20)]

= Rs. 11109. .

:. C.I. = Rs. (11109 - 8000) = Rs. 3109.

Ex. 3. ***Find the compound interest* on *Rs. 10,000 in* 2 *years* at 4% *per* annum, *the interest being compounded half-yearly.*** **(S.S.C. 2000)**

Sol.

Principal = Rs. 10000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

Amount =
Rs [10000 * (1+(2/100))^4] = Rs(10000 * (51/50) * (51/50) * (51/50) * (51/50))

= Rs. 10824.32.

:. C.I. = Rs. (10824.32 - 10000) = Rs. 824.32.

Ex. 4. ***Find the compound interest* on *Rs. 16,000* at *20% per* annum *for* 9 *months, compounded quarterly.***

Sol. Principal = Rs. 16000; Time = 9 months =3 quarters;
Rate = 20% per annum = 5% per quarter.

Amount = Rs. [16000 x (1+(5/100))^3] = Rs. 18522.

CJ. = Rs. (18522 - 16000) = Rs. 2522.

Ex. 5. ***If the simple interest*** **on a** ***sum*** **of** ***money*** **at 5%** ***per*** **annum** ***for*** **3** ***years is Rs. 1200, find the compound interest*** **on** ***the same sum for the*** **same** ***period at the same r*****ate.**

Sol.

Clearly, Rate = 5% p.a., Time = 3 years, S.I.= Rs. 1200. . .

So principal=RS [100*1200]/3*5=RS 8000

Amount = Rs. 8000 x [1 +5/100]^3 - = Rs. 9261.

.. C.I. = Rs. (9261 - 8000) = Rs. 1261.

Ex. 6. ***In what time will Rs. 1000 become Rs.*** **1331 at** ***10% per*** **annum** ***compounded annually?*** **(S.S.C. 2004)**

Sol.

Principal = Rs. 1000; Amount = Rs. 1331; Rate = 10% p.a. Let the time be n years. Then,
[1000 $(1+ (10/100))^n$] = 1331 or $(11/10)^{n}$ = (1331/1000) = $(11/10)^3$
n = 3 years.

Ex. 7. ***If Rs. 600 amounts*** **to** ***Rs. 683.20 in*** **two years** ***compounded*** **annually, find the rate of** ***interest per*** **annum.**

Sol. Principal = Rs. 500; Amount = Rs. 583.20; Time = 2 years.
Let the rate be R% per annum.. 'Then,

[500 $(1+(R/100)^2$] = 583.20 or [1+ $(R/100)]^2$ = 5832/5000 = 11664/10000

[1+ $(R/100)]^2 = (108/100)^2$ or 1 + (R/100) = 108/100 or R = 8

So, rate = 8% p.a.

Ex. 8. ***If the compound interest*** **on a** ***certain sum*** **at 16 (2/3)%** ***to*** **3** ***years is Rs.1270, find the simple interest*** **on** ***the same sum*** **at** ***the same*** **rate and** ***f or the*** **same period.**

Sol. Let the sum be Rs. x. Then,

C.I. = [x * (1 + (($50/(3*100))^3$ - x] = ((343x / 216) - x) = 127x / 216

127x /216 = 1270 or x = (1270 * 216) / 127 = 2160.

Thus, the sum is Rs. 2160

S.I. = Rs (2160 * (50/3) * 3 * (1 /100)) = Rs. 1080.

Ex. 9. ***The difference*** **between** ***the compound interest*** **and** ***simple interest*** **on a** ***certain sum*** **at 10%** ***per*** **annum** ***for*** **2** ***years is Rs.*** **631.** ***Find the sum.***

Sol. Let the sum be Rs. x. Then,

C.I. = $x (1 + (10 /100))^2 - x = 21x / 100$,

S.I. = $((x * 10 * 2) / 100) = x / 5$

$(C.I) - (S.I) = ((21x / 100) - (x / 5)) = x / 100$

$(x / 100) = 632$ □ $x = 63100$.

Hence, the sum is Rs.63,100.

Ex. 10. ***The difference* between *the compound interest* and *the simple* interest *accrued* on an amount *of Rs. 18,000 in* 2 *years was Rs. 405. What* was *the* rate *of interest p.c.p.a.* ?** **(Bank P.O. 2003)**

Sol. Let the *rate* be R% p.a. then,

$[18000 (1 + (R / 100)^2) - 18000] - ((18000 * R * 2) / 100) = 405$

$18000 [(100 + (R / 100)^2 / 10000) - 1 - (2R / 100)] = 405$

$18000[((100 + R)^2 - 10000 - 200R) / 10000] = 405$

$9R^2 / 5 = 405$ □ $R^2 =((405 * 5) / 9) = 225$

$R = 15$.

Rate = 15%.

Ex. 11. ***Divide Rs. 1301 between A and B, so that the amount* of *A after* 7 *years is equal to the amount* of *B after* 9 *years, the interest being compounded at* 4% *per annum.***

Sol. Let the two parts be Rs. x and Rs. (1301 - *x).*

$x(1+4/100)^7 =(1301-x)(1+4/100)^9$

$x/(1301-x)=(1+4/100)^2=(26/25*26/25)$

$625x=676(1301-x)$

$1301x=676*1301$

$x=676$.

So,the parts are rs.676 and rs.(1301-676)i.e rs.676 and rs.625.

Ex.12. a certain sum amounts to rs.7350 in 2 years and to rs.8575 in 3 years.find the sum and rate percent.

S.I on rs.7350 for 1 year=rs.(8575-7350)=rs.1225.

*Rate=(100*1225/7350*1)%=16 2/3%*

Let the sum be rs.x.then,

$X(1+50/3*100)^2=7350$

$X*7/6*7/6=7350$

$X=(7350*36/49)=5400$.

Sum=rs.5400.

Ex.13.a sum of money amounts to rs.6690 after 3 years and to rs.10,035 after 6 years on compound interest.find the sum.

Sol. *Let the sum be rs.P.then*

$P(1+R/100)^3=6690\ldots(i)$ and $P(1+R/100)^6=10035\ldots(ii)$

On dividing,we get $(1+R/100)^3=10025/6690=3/2$.

Substituting this value in (i),we get:

$P*3/2=6690$ or $P=(6690*2/3)=4460$

Hence,the sum is rs.4460.

Ex.14. a sum of money doubles itself at compound interest in 15 years.in how many years will it beco,e eight times?

$P(1+R/100)^{15}=2P$

$(1+R/100)^{15}=2P/P=2$

LET $P(1+R/100)^n=8P$

$(1+R/100)^n=8=2^3=\{(1+R/100)^{15}\}^3$ [USING (I)]

$(1+R/100)^N=(1+R/100)^{45}$

n=45.

Thus,the required time=45 years.

Ex.15.What annual payment will discharge a debt of Rs.7620 due in 3years at 16 2/3% per annum interest?

Sol. Let each installment beRs.x.

Then,(P.W. of Rs.x due 1 year hence)+(P>W of Rs.x due 2 years hence)+(P.W of Rs. X due 3 years hence)=7620.

$\therefore$ $x/(1+(50/3*100))+ x/(1+(50/3*100))^2 + x/(1+(50/3*100))^3=7620$

$\Leftrightarrow(6x/7)+(936x/49)+(216x/343)=7620.$

$\Leftrightarrow 294x+252x+216x=7620*343.$

$\Leftrightarrow$ x=(7620*343/762)=3430.

$\therefore$ Amount of each installment=Rs.3430.

23. LOGARITHMS

IMPORTANT FACTS AND FORMULAE

I. **Logarithm**: If a is a positive real number, other than 1 and $a^m = X$, then we write:
m = $\boldsymbol{log_a}$ **x** and we say that the value of log x to the base *a* is m.

Example:

(i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(ii) $2^{-3} = 1/8 \Rightarrow \log_2 1/8 = -3$
(iii) $3^4 = 81 \Rightarrow \log_3 81 = 4$
(iiii) $(.1)^2 = .01 \Rightarrow \log_{(.1)} .01 = 2$.

II. Properties of Logarithms:

1. $\log_a(xy) = \log_a x + \log_a y$

2. $\log_a (x/y) = \log_a x - \log_a y$

3. $\log_x x = 1$

4. $\log_a 1 = 0$

5. $\log_a(x^p) = p(\log_a x)$ 1
6. $\log_a x = 1/\log_x a$
7. $\log_a x = \log_b x/\log_b a = \log\_x/\log a$.

Remember: When base is not mentioned, it is taken as 10.

II. **Common Logarithms:**
Logarithms to the base 10 are known as common logarithms.

III. The logarithm of a number contains two parts, namely *characteristic* and *mantissa.*
Characteristic: The integral part of the logarithm of a number is called its ***characteristic.***

Case I: When the number is greater than 1.
In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II: When the number is less than 1.
In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.
Instead of - 1, - 2, etc. we write, $\bar{1}$ (one bar), $\bar{2}$ (two bar), etc.

Example:

Number	*Characteristic*	*Number*	*Characteristic*
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa: The decimal part of the logarithm of a number is known is its ***mantissa.*** For mantissa, we look through log table.

SOLVED EXAMPLES

1.Evaluate:

(1)**log**$_3$ 27

(2)**log**$_7$ (1/343)

(3)**log**$_{100(0.01)}$

SOLUTION:

(1) let $\log_3 27 = 3^3$ or n=3.

ie, $\log_3 27 = 3$.

(2) Let $\log_7 (1\backslash 343) = n$.

Then ,$7^n = 1/343$

$= 1/7^3$

n = -3.

ie,

$\log_7(1\backslash 343) = -3$.

(3) let $\log_{100}(0.01) = n$.

Then,. (100) = 0.01 = 1 /100=100 $^{-1}$ 0r n=-1

EX.2. evaluate

(i) $\log_7 1=0$ (ii)$\log_{34} 34$ (iii)$36^{\log_6 4}$

solution:

i) we know that $\log_a 1=0$,so $\log_7 1=0$.

ii) we know that $\log_a a=1$,so $\log_{34} 34=0$.

iii) We know that $a^{\log_6 x} = x$.

· now $36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{\log_6 (16)=16.}$

Ex.3.if $\log_{\sqrt{8}} x=3\ (1/3)$, find the value of x.

$\log_{\sqrt{8}} x=10/3$,$x=(\sqrt{8})^{10/3}=(2^{3/2})^{10/3}=2^{(3/2*10/3)}=2^5=32$.

Ex.4:Evaluate: (i) $\log_5 3*\log_{27} 25$ (ii) $\log 27 - \log_{27} 9$

(i)$\log_5{}^3 * \log_{27} 25=(\log 3/\log 5)*(\log 25/\log 27)$
$=(\log 3/\log 5)*(\log 5^2*\log 3^3)$
$=(\log 3/\log 5)*(2\log 5/3\log 3)$
$=2/3$

(ii)Let $\log_9 27=n$
Then,
$9^n=27 \Leftrightarrow 3^{2n}=3^3 \Leftrightarrow 2n=3 \Leftrightarrow n=3/2$
Again, let $\log_{27} 9=m$
Then,
$27^m=9 \Leftrightarrow 3^{3m}=3^2 \Leftrightarrow 3m=2 \Leftrightarrow m=2/3$

- $\Rightarrow$ $\log_9 27-\log_{27} 9=(n-m)=(3/2-2/3)=5/6$

Ex 5. Simplify :(log 75/16-2 log 5/9+log 32/243)
Sol: log 75/16-2 log 5/9+log 32/243
= log 75/16-log$(5/9)^2$+log32/243
= log 75/16-log25/81+log 32/243
= log(75/16*32/243*81/25)=log 2

Ex. 6.Find the value of x which satisfies the relation
$\text{Log}_{10} 3+\log_{10}(4x+1)=\log_{10}(x+1)+1$
Sol: $\log_{10} 3+\log_{10}(4x+1)=\log_{10}(x+1)+1$
$\text{Log}_{10} 3+\log_{10}(4x+1)=\log_{10}(x+1)+\log_{10}(x+1)+\log_{10} 10$
$\text{Log}_{10}(3(4x+1))=\log_{10}(10(x+1))$
$=3(4x+1)=10(x+1)=12x+3$
$=10x+10$
$=2x=7=x=7/2$

Ex. 7.Simplify:[$1/\log_{xy(}xyz)+1/\log_{yz}(xyz)+1/\log_{zx}(xyz)$]
Given expression: $\log_{xyz} xy+\log_{xyz} yz+\log_{xyz} zx$
$=\log_{xyz}(xy*yz*zx)=\log_{xyz}(xyz)^2$
$2\log_{xyz}(xyz)=2*1=2$

Ex.8.If $\log_{10} 2=0.30103$,find the value of $\log_{10} 50$.
Soln. $\log_{10} 50=\log_{10}(100/2)=\log_{10} 100-\log_{10} 2=2-0.30103=1.69897$.

Ex 9.If log 2=0.3010 and log 3=0.4771,find the values of:

i) log 25 ii)log 4.5

Soln.

i) log 25=log(100/4)=log 100-log 4=2-2log 2=(2-2*.3010)=1.398.

ii) log 4.5=log(9/2)=log 9-log 2=2log 3-log 2
=(2*0.4771-.3010)=.6532

Ex.10. If log 2=.30103,find the number of digits in 2^{56}.

Soln. log 2^{56} =56log2=(56*0.30103)=16.85768.

Its characteristics is 16.

Hence,the number of digits in 2^{56} is 17

24 .AREA

FUNDEMENTAL CONCEPTS

I.RESULTS ON TRIANGLES:
1.Sum of the angles of a triangle is 180 degrees.

2.Sum of any two sides of a triangle is greater than the third side.

3.Pythagoras theorem:
In a right angle triangle,
(Hypotenuse)^2 = (base)^2 + (Height)^2

4.The line joining the midpoint of a side of a triangle to the opposite vertex is called the

MEDIAN
5.The point where the three medians of a triangle meet is called **CENTROID**.
Centroid divides each of the medians in the ratio 2:1.

6.In an isosceles triangle, the altitude from the vertex bi-sects the base

7.The median of a triangle divides it into two triangles of the same area.

8.Area of a triangle formed by joining the midpoints of the sides of a given triangle is one-fourth of the area of the given triangle.

II.RESULTS ON QUADRILATERALS:

1. The diagonals of a parallelogram bisects each other .
2. Each diagonal of a parallelogram divides it into two triangles of the same area
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelograms of a given sides , the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

I.1.Area of a rectangle=(length*breadth)
Therefore length = (area/breadth) and breadth=(area/length)
2.Perimeter of a rectangle = 2*(length+breadth)
II.Area of a square = (side)^2 =1/2(diagonal)^2
III Area of four walls of a room = 2*(length + breadth)*(height)
IV 1.Area of the triangle=1/2(base*height)
2. Area of a triangle = (s*(s-a)(s-b)(s-c))^(1/2), where a,b,c are the sides of a triangle

and s= ½(a+b+c)

3.Area of the equilateral triangle =((3^1/2)/4)*(side)^2

4.Radius of incircle of an equilateral triangle of side a=a/2(3^1/2)

5.Radius of circumcircle of an equilateral triangle of side a=a/(3^1/2)

6.Radius of incircle of a triangle of area del and semiperimeter S=del/S

V.1.Area of the parellogram =(base *height)

2.Area of the rhombus=1/2(product of the diagonals)

3.Area of the trapezium=1/2(size of parallel sides)*distance between them

VI 1.Area of a circle =pi*r^2,where r is the radius

2. Circumference of a circle = 2∏R.

3. Length of an arc = 2∏Rθ/(360) where θ is the central angle

4. Area of a sector = (1/2) (arc x R) = pi*R^2*θ/360.

VII. 1. Area of a semi-circle = (pi)*R^2.

2. Circumference of a semi-circle = (pi)*R.

SOLVED EXAMPLES

Ex.1. One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.

Sol. Other side = $((17)^2 - (15)^{2)(1/2)}) = (289- 225)^{(1/2)} = (64)^{(1/2)} = 8$ m.

Area = (15 x 8) m^2 = 120 m^2.

Ex. 2. A lawn is in the form of a rectangle having its sides in the ratio 2: 3. The area of the lawn is (1/6) hectares. Find the length and breadth of the lawn.

Sol. Let length = 2x metres and breadth = 3x metre.

Now, area = (1/6)x 1000 m^2 = 5000/3m^2

So, 2x * *3x* = 5000/3 <=> x^2 = 2500/9 <=> *x* = 50/3

therefore Length = 2x = (100/3) m = 33(1/3) m and Breadth = *3x* = 3(50/3) m = 50m.

Ex. 3. Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.

Sol. Area of the carpet = Area of the room = (13 * 9) m^2 = 117 m^2.

Length of the carpet = (area/width) = 117 *(4/3) m = 156 m.

Therefore Cost of carpeting = Rs. (156 * 12.40) = Rs. 1934.40.

Ex. 4. If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle. .

Sol. Let length = x and breadth = *y*. Then,

$2(x + y) = 46$ or $x + y = 23$ and $x^2 + y^2 = (17)^2 = 289$.

Now, $(x + y)^2 = (23)^2 \Leftrightarrow (x^2 + y^2) + 2xy = 529 \Leftrightarrow 289 + 2xy = 529 \Leftrightarrow xy=120$

Area = xy = 120 cm^2.

Ex. 5. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.

Sol. Let breadth = x. Then, length = *2x.* Then,
$(2x - 5)(x + 5) - 2x * x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.
:. Length of the rectangle = 20 cm.

Ex. 6. In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements. (M.B.A. 2003)
Sol. Let x and *y* be the sides of the rectangle. Then, Correct area = xy.
Calculated area = (105/100)*x * (96/100)*y = (504/500)(xy)
Error In measurement = (504/500)xy- *xy* = (4/500)xy
Error % = [(4/500)*xy* *(1/xy) *100] % = (4/5) % = 0.8%.

Ex. 7. A rectangular grassy plot 110 m. by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.

Sol. Area of the plot = $(110 \times 65)\ m^2 = 7150\ m^2$
Area of the plot excluding the path = $[(110 - 5) * (65 - 5)]\ m^2 = 6300\ m^2$.
Area of the path = $(7150 - 6300)\ m^2 = 850\ m^2$.

Cost of gravelling the path = Rs.850 * (80/100)= Rs. 680

Ex. 8. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares. (S.S.C. 2003)
Sol. Side of first square = (40/4) = 10 cm;
Side of second square = (32/4)cm = 8 cm.
Area of third square = $[(10)^2 - (8)^2]\ cm^2 = (100 - 64)\ cm^2 = 36\ cm^2$.
Side of third square = $(36)^{(1/2)}$ cm = 6 cm.
Required perimeter = (6 x 4) cm = 24 cm.

Ex. 9. A room 5m 55cm long and 3m 74 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.
Sol. Area of the room = $(544 \times 374)\ cm^2$.
Size of largest square tile = H.C.F. of 544 cm and 374 cm = 34 cm.
Area of 1 tile = $(34 \times 34)\ cm^2$.
Number of tiles required =(544*374)/(34*34)=176

Ex. 10. Find the area of a square, one of whose diagonals is 3.8 m long.
Sol. Area of the square = $(1/2)*$ (diagonal) ^{2} = $[(1/2)*3.8*3.8\ ]m^2 = 7.22\ m^2$.

Ex. 11. The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas. (Section Officers', 2003)
Sol. Let the diagonals of the squares be 2x and 5x respectively.

Ratio of their areas = $(1/2)*(2x)^2 : (1/2)*(5x)^2 = 4x^2 : 25x^2 = 4 : 25$.

Ex.12. If each side of a square is increased by 25%, find the percentage change in its area.

Sol. Let each side of the square be *a*. Then, area = a^2.

New side =(125a/100) =(5a/4). New area = $(5a/4)^2 = (25a^2)/16$.
Increase in area = $((25\ a^2)/16) - a^2 = (9a^2)/16$.
Increase% = $[((9a^2)/16)*(1/a^2)*100]$ % = 56.25%.

Ex. 13. If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.

Sol. Let x and y be the length and breadth of the rectangle respectively.
Then, x - 4 = y + 3 or x - y = 7 ----(i)
Area of the rectangle =xy; Area of the square = (x - 4) (*y* + 3)
(x - 4) (y + 3) =xy <=> 3x - 4*y* = 12 ----(ii)
Solving *(i)* and *(ii),* we get x = 16 and *y* = 9.
Perimeter of the rectangle = 2 (x + *y)* = [2 (16 + 9)] cm = 50 cm.

Ex. 14. A room is half as long again as it is broad. The cost of carpeting the at Rs. 5 per sq. m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m^2 is Rs. 1720. If a door and 2 windows occupy 8 sq. m, find the dimensions of the room.

Sol. Let breadth = x metres, length = <u>*3x* </u>metres, height = H metres.

Area of the floor=(Total cost of carpeting)/(Rate/m^2)=(270/5)m^2=54m^2.
*x** (3x/2) = 54 <=> *x*^2 = (54*2/3) = 36 <=> *x* = 6.
So, breadth = 6 m and length =(3/2)*6 = 9 m.
Now, papered area = (1720/10)m^2 = 172 m^2.
Area of 1 door and 2 windows = 8 m^2.
Total area of 4 walls = (172 + 8) m^2 = 180 m^2

2*(9+ 6)* H = 180 <=> H = 180/30 = 6 m.

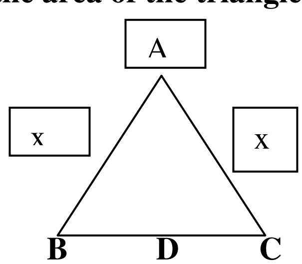
Ex. 15. Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.

Sol. Let a = 13, b = 14 and *c* = 15. Then, *S* = (1/2)*(a* + *b* + c) = 21.
(s- a) = 8, (s - b) = 7 and (s - *c)* = 6.
Area = $(s(s-a)(s-b)(s-c))^{(1/2)} = (21 * 8 * 7 * 6)^{(1/2)} = 84\ cm^2$.

Ex. 16. Find the area of a right-angled triangle whose base is 12 cm and hypotenuse is 13cm.
Sol. Height of the triangle = $[(13)^2 - (12)^2]^{(1/2)}$ cm = $(25)^{(1/2)}$ cm = 5 cm.
Its area = (1/2)* Base * Height = ((1/2)*12 * 5) cm^2 = 30 cm^2.

Ex. 17. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.
Sol. Area of the field = Total cost/rate = (333.18/25.6)hectares = 13.5 hectares
⇔ (13.5 x 10000) m^2 = 135000 m^2.
Let altitude = x metres and base = 3x metres.
Then, (1/2)* *3x* * *x* = 135000 <=>x^2= 90000 <=>*x* = 300.
Base = 900 m and Altitude = 300 m.

Ex. 18. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.



Sol. Let ABC be the isosceles triangle and AD be the altitude.
Let AB = AC = x. Then, BC = (32 - *2x).*
Since, in an isosceles triangle, the altitude bisects the base,
so BD = DC = (16 - *x).*
In *triangle* ADC, AC^2= AD + DC^2=>x^2=(8^2)+$(16-x)^2$
=>*32x* = 320 =>x= 10.
BC = (32- *2x)* = (32 - 20) cm = 12 cm.
Hence, required area = ((1/2)x*BC * AD) = ((1/2)*12 *10)cm^2 = 60 cm^2.

Ex. 19. Find the length of the altitude of an equilateral triangle of side 3√3 cm.

Sol. Area of the triangle = (√3/4) x *(3 √3)2* = 27 √3. Let the height be *h.*

Then, (1/2) x 3√3 x *h* = (27√3/4) X(2/√3) = 4.5 cm.

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Ex. 20. In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.

Sol. Let the bases of the two triangles be x and y and their heights be *3h* and *4h* respectively. Then,

((1/2) X x X 3h)/(1/2) X y X 4h) =4/3 ⇔ x/y =(4/3 X 4/3)=16/9

Required ratio = 16 : 9.

Ex.21. The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.

Sol. Let the height of the parallelogram be x. cm. Then, base = (2x*)* cm.

2x X x =72 ⇔ 2x^2 = 72 ⇔ X ^2=36 ⇔ x=6

Hence, height of the parallelogram = 6 cm.

Ex. 22. Find the area of a rhombus one side of which measures 20 cm and 01 diagonal 24 cm.

Sol. Let other diagonal = 2x cm.

Since diagonals of a rhombus bisect each_other at right angles, we have:

(20)2 = (12)2 + (x)2 _ *x* = √(20)2 – (12)2= √256= 16 cm. _I

So, other diagonal = 32 cm.

Area of rhombus = (1/2) x (Product of diagonals) = *((1/2)x* 24 x 32) cm^2 = 384 cm^2

Ex. 23. The difference between two parallel sides of a trapezium is 4 cm. perpendicular distance between them is 19 cm. If the area of the trapezium is 475 find the lengths of the parallel sides. (R.R.B. 2002)

Sol. Let the two parallel sides of the trapezium be *a* em and b em.

Then, *a* - *b* = 4

And, (1/2) x *(a + b)* x 19 = 475 ⇔ *(a + b)* =((475 x 2)/19) ⇔ *a* + *b* = 50

Solving (i) and *(*ii*),* we get: *a* = 27, *b* = 23.

So, the two parallel sides are 27 cm and 23 cm.

Ex. 24. Find the length of a rope by which a cow must be tethered in order tbat it may be able to graze an area of 9856 sq. metres. (M.A.T. 2003)

Sol. Clearly, the cow will graze a circular field of area 9856 sq. metres and radius equal to the length of the rope.

Let the length of the rope be R metres.

Then, ∏(R)^2 = (9856 X (7/22)) = 3136⇔ R = 56.

Length of the rope = 56 m.

Ex. 25. The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.

Sol. Area = (13.86 x 10000) m^2= 138600 m^2.

∏(R^2= 138600 ⇔(R)2 = (138600 x (7/22)) ⇔ R = 210 m.

Circumference = 2∏R = (2 x (22/7) x 210) m = 1320 m.
Cost of fencing = Rs. (1320 x 4.40) = Rs. 5808.

Ex. 26. The diameter of the driving wheel of a bus is 140 em. How many revolution, per minute must the wheel make in order to keep a speed of 66 kmph ?

Sol. Distance to be covered in 1 min. = (66 X_1000)/(60) m = 1100 m.

Circumference of the wheel = (2 x (22/7) x 0.70) m = 4.4 m.

Number of revolutions per min. =(1100/4.4) = 250.

Ex, 27. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

Sol. Distance covered in one revolution =((88 X 1000)/1000)= 88m.

2∏R = 88 ⇔ 2 x (22/7) x R = 88 ⇔ R = 88 x (7/44) = 14 m.

Ex, 28. The inner circumference of a circular race track, 14 m wide, is 440 m. Find radius of the outer circle.

Sol . Let inner radius be *r* metres. Then, 2∏r = 440 ⇔ r = (440 x (7/44))= 70 m.

Radius of outer circle = (70 + 14) m = 84 m.

Ex, 29. Two concentric circles form a ring. The inner and outer circumferences of ring are (352/7) m and (518/7) m respectively. Find the width of the ring.

Sol.. Let the inner and outer radii be r and R metres.

Then 2∏*r* = (352/7) ⇔ r =((352/7) X (7/22) X (1/2))=8m.

2∏R=(528/7) ⇔ R=((528/7) X (7/22) X (1/2))= 12m.

, ', Width of the ring = (R - *r)* = (12 - 8) m = 4 m.

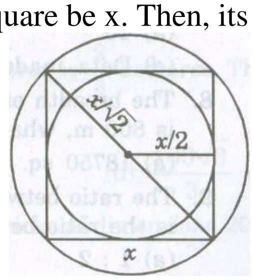
Ex, 30. A sector of 120', cut out from a circle, has an area of (66/7) sq. cm. Find the radius of the circle.

Sol. Let the radius of the circle be *r* cm. Then,
$(\prod(r)^2\,\theta)/360=(66/7) \Leftrightarrow (22/7) \times (r)^2 \times (120/360)= (66/7)$
$\Leftrightarrow (r)^2=((66/7) \times (7/22) \times 3) \Leftrightarrow r=3.$

Hence, radius = 3 cm.

Ex, 31. Find the ratio of the areas of the incircle and circumcircle of a square.

Sol. Let the side of the square be x. Then, its diagonal = $\sqrt{2}\,x$.



Radius of incircle = $(x/2)$
Radius of circum circle= $(\sqrt{2}x/2) =(x/\sqrt{2})$

Required ratio = $((\prod (r)^2)/4 : (\prod(r)^2)/2) = (1/4) : 1/2) = 1 : 2.$

Ex. 32. If the radius of a circle is decreased by 50%, find the percentage decrease in its area.

Sol. Let original radius = R. New radius $=(50/100)\,R$ = <u>$(R/2)$</u>

Original area$=\prod(R)^2=$ and new area$= \prod ((R/2))^2= (\prod (R)^2)/4$
Decrease in area $=((3\prod (R)^2)/4 \times (1/\prod(R)^2) \times 100)\ \% = 75\%$

25.VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = 1, breadth = b and height = h units. Then, 1. Volume = (1 x b x h) cubic units.

2. Surface area= 2(lb + bh + lh) sq.units.

3. Diagonal.=$\sqrt{l^2 + b^2 + h^2}$ units

II. CUBE

Let each edge of a cube be of length a. Then,

1. Volume = a^3 cubic units.

2. Surface area = $6a^2$ sq. units.

3. Diagonal = $\sqrt{3}$ a units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. Volume = ($\Pi r^2 h$) cubic units.

2. Curved surface area = (2Π rh). units.

3. Total surface area =$2\Pi r$ (h+r) sq. units

IV. CONE

Let radius of base = r and Height = h. Then,

1. Slant height, l =$\sqrt{h^2+r^2}$

2. Volume = (1/3) $\Pi r^2 h$ cubic units.

3. Curved surface area = (Πrl) sq. units.

4. Total surface area = ($\Pi rl + \Pi r^2$) sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. Volume = (4/3)Πr3 cubic units.

2. Surface area = ($4\Pi r^2$) sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. Volume = (2/3)Πr^3 cubic units.

2. Curved surface area = ($2\Pi r^2$) sq. units.

3. Total surface area = ($3\Pi r^2$) units.

Remember: 1 litre = 1000 cm3.

SOLVED EXAMPLES

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Ex. 1. Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.

Sol. Volume = (16 x 14 x 7) m3 = 1568 m3.

1 Surface area = [2 (16 x 14 + 14 x 7 + 16 x 7)] cm^2 = (2 x 434) cm^2 = 868 cm^2.

Ex. 2. Find the length of the longest pole that can be placed in a room 12 m long 8m broad and 9m high.

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Sol. Length of longest pole = Length of the diagonal of the room

= √(12^2+8^2+9^2= .√(289)= 17 m.

Ex. 3. Tbe volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.

Sol. Let the breadth of the wall be x metres.

Then, Height = 5x metres and Length = 40x metres.

∴x * 5x * 40x = 12.8 ⇔ x^3=12.8/200 = 128/2000 = 64/1000

So, x = (4/10) m =((4/10)*100)cm = 40 cm

Ex. 4. Find the number of bricks, each measuring 24 cm x 12 cm x 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?

Sol. Volume of the wall = (2400 x 800 x 60) cu. cm.

Volume of bricks = 90% of the volume of the wall

=((90/100)*2400 *800 * 60)cu.cm.

Volume of 1 brick = (24 x 12 x 8) cu. cm.

∴Number of bricks=(90/100)*(2400*800*60)/(24*12*8)=45000.

Ex. 5. Water flows into a tank 200 m x 160 m througb a rectangular pipe of 1.5m x 1.25 m @ 20 kmph . In what time (in minutes) will the water rise by 2 metres?

Sol. Volume required in the tank = (200 x 150 x 2) m^3 = 60000 m^3.

. .

Length of water column flown in1 min =(20*1000)/60 m =1000/3 m

Volume flown per minute = 1.5 * 1.25 * (1000/3) m^3 = 625 m^3.

∴ Required time = (60000/625)min = 96min

Ex. 6. Tbe dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 2 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.

Sol. Volume of the metal used in the box = External Volume - Internal Volume

= [(50 * 40 * 23) - (44 * 34 * 20)]cm^3

= 16080 cm^3

∴ Weight of the metal =((16080*0.5)/1000) kg = 8.04 kg.

Ex. 7. The diagonal of a cube is 6√3cm. Find its volume and surface area.

Sol. Let the edge of the cube be a.

$\therefore \sqrt{3}a = 6../3$ _ a = 6.

So,Volume = a^3 = (6 x 6 x 6) cm^3 = 216 cm^3.

Surface area = $6a^2$ = (6 x 6 x 6) cm^2 == 216 cm^2.

Ex. 8. The surface area of a cube is 1734 sq. cm. Find its volume.

Sol. Let the edge of the cube bea. Then,

$6a^2 = 1734 \Rightarrow a^2 = 289$ => a = 17 cm.

$\therefore$ Volume = $a^3 = (17)^3$ cm^3 = 4913 cm^3.

Ex. 9. A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.

Sol. Volume of the block = (6 x 12 x 15) cm^3 = 1080 cm^3.

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm = 3 cm.

Volume of this cube = (3 x 3 x 3) cm^3 = 27 cm^3.

Number of cubes = 1080/27 = 40.

Ex.l0. A cube of edge 15 cm is immersed completely in a rectangular vessel containing water . If the dimensions of the base of vessel are 20 cm x 15 cm, find the rise in water level.

Sol. Increase in volume = Volume of the cube = (15 x 15 x 15) cm^3.

$\therefore$Rise in water level = volume/area = (15 x 15 x 15)/(20 x 15) cm = 11.25 cm.

Ex. 11. Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.

Sol. Volume of new cube = $(1^3 + 6^3 + 8^3)$ cm+ = 729 cm^3.

Edge of new cube = $_3\sqrt{729}$ cm = 9 cm.

$\therefore$ Surface area of the new cube = (6 x 9 x 9) cm^2 = 486 cm^2.

Ex. 12. If each edge of a cube is increased by 50%, find the percentage increase in
Its surface area. "

Sol. Let original length of each edge = a.

Then, original surface area = $6a^2$.

New edge = (150% of a) = (150a/100) = 3a/2

New surface area = $6x\,(3a/2)^2 = 27a^2/2$

Increase percent in surface area = $\left(\left(\frac{15a^2}{2}\right) \times \left(\frac{1}{6a^2}\right) \times 100\right)\% = 125\%$

Ex. 13. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.

Sol. Let their edges be a and b. Then,

$a^3/b^3 = 1/27$ (or) $(a/b)^3 = (1/3)^3$ (or) $(a/b) = (1/3)$.

$\therefore$ Ratio of their surface area = $6a^2/6b^2 = a^2/b^2 = (a/b)^2 = 1/9$, i.e. 1:9.

Ex.14.Find the volume , curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.

Sol. Volume = $\prod r^2 h = ((22/7)x(7/2)x(7/2)x40) = 1540\ cm^{3.}$.

Curved surface area = $2\prod rh = (2x(22/7)x(7/2)x40) = 880\ cm^2$.

Total surface area = $2\prod rh + 2\prod r^2 = 2\prod r\,(h + r)$

$= (2 \times (22/7) \times (7/2) \times (40+3.5))\ cm^2$

$= 957\ cm^2$

Ex.15. If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.

Sol. Let the depth of the tank be h metres. Then,

$\prod \times 7^2 \times h = 1848 \Rightarrow h = (1848 \times (7/22) \times (1/49) = 12$ m

Ex.16. 2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm **diameter. Find the length of the wire in metres.**

Sol. Let the length of the wire be h metres. Then,

$\prod\ (0.50/(2 \times 100))^2 \times h = 2.2/1000$

$\Rightarrow$ h = ((2.2/1000) x (100 x 100)/(0.25 x 0.25) x (7/22)) = 112 m.

Ex. 17. How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron?

Sol. Volume of 1 rod = ((22/7) x (1/100) x (1/100) x 7) cu.m = 11/5000 cu.m

Volume of iron = 0.88 cu. m.

Number of rods = (0.88 x 5000/11) = 400.

Ex. 18. The radii of two cylinders are in the ratio 3: 5 and their heights are in tbe ratio of 2 : 3. Find the ratio of their curved surface areas.

Sol. Let the radii of the cylinders be 3x, 5x and their heights be 2y, 3y respectively. Then
Ratio of their curved surface area = $\frac{2\prod X\ 3x\ X\ 2y}{2\prod X\ 5x\ X\ 3y}$ = 2/5 = 2.5

Ex. 19. If 1 cubic cm of cast iron weighs 21 gms, then find the eight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 em.

Sol. Inner radius = (3/2) cm = 1.5 cm, Outer radius = (1.5 + 1) = 2.5 cm.

$\therefore$ Volume of iron = $[\prod \times (2.5)^2 \times 100 - \prod \times (1.5)^2 \times 100]\ cm^3$

$= (22/7) \times 100 \times [(2.5)^2 - (1.5)^2]\ cm^3$

$= (8800/7)\ cm^3$

Weight of the pipe = ((8800/7) x (21/1000))kg = 26.4 kg.

Ex. 20. Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.

Sol. Here, r = 21 cm and h = 28 cm.
$\therefore$ Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35cm$

Ex. 21. Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.

Sol. Here, r = 7m and h = 24 m.

So,$l = \sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(625)} = 25$ m.
Area of canvas = $\prod rl=((22/7)*7*25)m^2 = 550\ m^2$.
Length of canvas = (Area/Width) = (550/1.25) m = 440 m.

Ex. 22. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.

Sol. Let the radii of their bases be r and R and their heights be h and 2h respectively.
Then,$(2\prod r/2\prod R)=(3/4) \Rightarrow R=(4/3)r$.
$\therefore$ Ratio of volumes = $(((1/3)\prod r^2h)/((1/3)\prod(4/3r)^2(2h)))=9 : 32$.

Ex. 23. The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and It heights are in the ratio 2 : 3. Find the ratio of their volumes.

Sol. Let the radii of the cylinder and the cone be 3r and 4r and their heights be 2h and 3h respectively.

$\therefore \frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\prod \times (3r)^2 * 2h}{(1/3)\prod r^2 * 3h} = 9/8 = 9 : 8$.

Ex. 24. A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of

liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.

Sol. Volume of the liquid in the cylindrical vessel

= Volume of the conical vessel

= ((1/3)* (22/7)* 12 * 12 * 50))cm^3 = (22 *4 *12 * 50)/7 cm^3.

Let the height of the liquid in the vessel be h.

Then (22/7)*10*10*h =(22*4*12*50)/7 or h = (4*12*50)/100 = 24 cm

Ex. 25. Find the volume and surface area of a sphere of radius 10.5 cm.

3 3

Sol. Volume = (4/3)∏r^3 =(4/3)*(22/7)*(21/2)*(21/2)*(21/2) cm^3 = 4851 cm^3.

Surface area = 4∏r^2 =(4*(22/7)*(21/2)*(21/2)) cm^2 = 1386 cm^2

Ex. 26. If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area.

Sol. Let original radius = R. Then, new radius = (150/100)R=(3R/2)

Original volume = (4/3)∏R^3, New volume = (4/3)∏$(3R/2)^3$ =(9∏R^3/2)

Increase % in volume=((19/6)∏R^3)*(3/4∏R^3)*100))% = 237.5%

Original surface area =4∏R^2. New surface area = 4∏$(3R/2)^2$=9∏R^2

Increase % in surface area =(5∏R^2/4∏R^2) * 100) % = 125%.

Ex. 27. Find the number of lead balls, each 1 cm in diameter that can be a sphere of diameter 12 cm.

Sol. Volume of larger sphere = (4/3)∏*6*6*6) cm^3 = 288∏ cm^3.

Volume of 1 small lead ball = ((4/3)∏*(1/2)*(1/2)*(1/2)) cm^3 = ∏/6 cm^3.

∴ Number of lead balls = (288∏*(6/∏)) = 1728.

Ex.28.How many spherical bullets can be made out of a lead cylinder 28cm high and with radius 6 cm, each bullet being 1.5 cm in diameter ?

Sol. Volume of cylinder = (∏ x 6 x 6 x 28) cm^3 = (9∏/16) cm^3.

Number of bullet = $\frac{\text{Volume of cylinder}}{\text{Volume of each bullet}}$ = [(36 x 28)∏ x 16] /9∏ = 1792.

Ex.29.A copper sphere of diameter 18cm is drawn into a wire of diameter 4 mm Find the length of the wire.

Sol. Volume of sphere = ((4∏/3) x 9 x 9 x 9) cm^3 = 972∏ cm^3

Volume of sphere = (∏ x 0.2 x 0.2 x h) cm^3

∴ 972∏ = ∏ x (2/10) x (2/10) x h ⇒ h = (972 x 5 x 5)cm = [(972 x 5 x5)/100] m

= 243m

Ex.30.Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a

sphere. Find the diameter of the sphere.

Sol. Volume of sphere = Volume of 2 cones

$= \left(\frac{1}{3}\pi \times (2.10^2) \times 4.1 + \frac{1}{3}\pi \times (2.1)^2 \times 4.3\right)$

Let the radius of sphere be R

$\therefore (4/3)\pi R^3 = (1/3)\pi(2.1)^3$ or $R = 2.1$cm

Hence , diameter of the sphere = 4.2.cm

Ex.31.A Cone and a sphere have equal radii and equal volumes. Find the ratio of the sphere of the diameter of the sphere to the height of the cone.

Sol. Let radius of each be R and height of the cone be H.

Then, $(4/3)\pi R^3 = (1/3)\pi R^2H$ (or) $R/H = ¼$ (or) $2R/H = 2/4 = 1/2$

$\therefore$Required ratio = 1:2.

Ex.32.Find the volume , curved surface area and the total surface area of a hemisphere of radius 10.5 cm.

Sol. Volume = $(2\pi r^3/3) = ((2/3) \times (22/7) \times (21/2) \times (21/2) \times (21/2))\text{cm}^3$

$= 2425.5 \text{ cm}^3$

Curved surface area = $2\pi r^3 = (2 \times (22/7) \times (21/2) \times (21/2))\text{cm}^2$

$=693 \text{ cm}^2$

Total surface area = $3\pi r^3 = (3 \times (22/7) \times (21/2) \times (21/2))\text{cm}^2$

$= 1039.5 \text{ cm}^{2.}$

Ex.33.Hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?

Sol. Volume of bowl = $((2\pi/3) \times 9 \times 9 \times 9)\text{ cm}^3 = 486\pi \text{ cm}^3$.

Volume of 1 bottle = $(\pi \times (3/2) \times (3/2) \times 4)\text{ cm}^3 = 9\pi \text{ cm}^3$

Number of bottles = $(486\pi/9\pi) = 54$.

Ex34.A Cone,a hemisphere and a cylinder stand on equal bases and have the same height.Find ratio of their volumes.

Sol. Let R be the radius of each

Height of the hemisphere = Its radius = R.

$\therefore$Height of each = R.

Ratio of volumes = $(1/3)\pi R^2 \times R : (2/3)\pi R^3 : \pi R^2 \times R = 1:2:3$

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26. RACES AND GAMES

IMPORTANT FACTS

Races: A contest of speed in running, riding, driving, sailing or rowing is called race

Course: The ground or path on which contests are made is called a race course.

Starting Point: The point from which a race begins is known as a starting point.

Winning Point or Goal: The point set to bound a race is called a winning paint or a goal.

Winner: The person who first reaches the winning point is called a winner.

Dead Heat Race: If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.

Start: Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that 'A gives B, a start of 12 metres. '
To cover a race of 100 metres in this case, A will have to cover 100 metres while B will
have to cover only (100 - 12) = 88 metres. i
In a 100 m race, 'A can give B 12 m' or 'A can give B a start of 12 m' or 'A beats 12 m' means that while A runs 100 m, B runs (100 - 12) = 88 m.

Games: 'A game of 100, means that the person among the contestants who scores 100m first is the winner.
If A scores 100 points while B scores only 80 points, then we say that 'A can give B 20 points.

SOLVED EXAMPLES :

Ex. 1. In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.

Sol. Clearly, B covers 28 m in 7 seconds.

:. B's time over the course = (278 x 1000) sec = 250 seconds.

:. A's time over the course = (250 - 7-) sec = 243 sec = 4 min. 3 sec.

Ex. 2. A runs 1 ¾ times as fast as B. if A gives B a start of 84 m, bow far must winning post be so that A and B might reach it at the same time?

Sol. Ratio of the rates of A and B = 7/4 : 1 = 7 : 4.
So, in a race of 7 m, A gains 3m over B.
:. 3 m are gained by A in a race of 7 m.

:. 84 m are gained by A in a race of (7/3 x 84) m = 196 m.

:. Winning post must be 196 m away from the starting point.

Ex. 3. A can run 1 km in 3 min. 10 sec. and B can cover the same distance in 3 min. 20 sec. By what distance can A beat B ?

Soln:Clearly, A beats B by 10 sec.

Distance covered by B in 10 sec. = $\left(\frac{1000}{200} \times 10\right)$m = 50 m.

Therefore A beats B by 50 metres.

Ex .4 . In a 100 m race, A runs at 8km per hour. If A gives B a start of 4 m and still him by 15 seconds, what is the speed of B ?

Sol: Time taken by A to cover 100 m =(60 X 60 / 8000) x 100 sec = 45 sec.
B covers (100 - 4) m = 96 m in (45 + 15) sec = 60 sec.
B's speed = $\left(\frac{96 \times 60 \times 60}{60 \times 1000}\right)$km/hr = 5.76 km/hr.

Ex. 5. A, Band C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64m how many metre's start can B give C ?

Sol: While A covers 1000 m, B covers (1000 - 40) m = 960 m and

C covers (1000 - 64) m or 936 m.
When B covers 960 m, C covers 936 m.

Ex 6. In a game of 80 points; A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60 ?

Sol. A: B = 80 : 75, A : C = 80 : 65.
B/C = (B/ A * A/C) = (75 / 80 * 80 / 65) = 15/13 = 60 /52 = 60: 5
Therfore ,In a game of 60, B can give C 8 points.

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27. CALENDAR

IMPORTANT FACTS AND FOAMULAE

Under this heading we mainly deal with finding the day of the week on a particular given date the process of finding it lies on obtaining the number of odd days.

Odd Days : Number of days more than the complete number of weeks in a given

Period ., is the number of odd days during that period.

LeapYear: Every year which is divisible by 4 is called a leap year.

Thus each one of the years 1992, 1996, 2004, 2008, 2012, etc. is a leap year. Every 4th century is a leap year but no other century is a leap year.thus each one of 400, 800, 1200,' 1600, 2000, etc. is a leap year.
None of 1900, 2010, 2020, 2100, etc. is a leap year.

An year which is not a leap year is called an ordinary year.

(I)An ordinary year has 365 days. (II) A leap year has 366 days.

Counting of Odd Days:

i)1 ordinary year = 365 days = (52 weeks + 1 day).
:. An ordinary year has 1 odd day.
ii)1 leap year = 366 days = (52 weeks + 2 days).
:. A leap year has 2 odd days.
iii)100 years = 76 ordinary years + 24 leap years
= [(76 x 52) weeks + 76 days) + [(24 x 52) weeks + 48 days]
= 5200 weeks + 124 days = (5217 weeks + 5 days).
:. 100 years contain 5 odd days.
200 years contain 10 and therefore 3 odd days.
300 years contain 15 and therefore 1 odd day.
400 years contain (20 + 1) and therefore 0 odd day.
Similarly, each one of 800, 1200, 1600, 2000, etc. contains 0 odd days.
Remark: (7n + m) odd days, where m < 7 is equivalent to m odd days.
Thus, 8 odd days ≡ 1 odd day etc.

No of odd days	0	1	2	3	4	5	6
Day	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.

SOLVED EXAMPLES

Ex: 1.Wbat was the day of the week on, 16th July, 1776?

Sol: 16th July, 1776 = (1775 years + Period from 1st Jan., 1776 to 16th July, 1776)
Counting of odd days :
1600 years have 0 odd day. 100 years have 5 odd days.
75 years = (18 leap years + 57 ordinary years)
= [(18 x 2) + (57 x 1)] odd days = 93 odd days
= (13 weeks + 2 days) = 2 odd days.

.. 1775 years have (0 + 5 + 2) odd days = 7 odd days = 0 odd day.
Jan. Feb. March April May June July
31 + 29 + 31 + 30 + 31 + 30 +16 = 198days
= (28 weeks + 2 days) =2days
:. . Total number of odd days = (0 + 2) = 2. Required day was 'Tuesday'.

Ex. 2. What was the day of the week on 16th August, 1947?

Sol. 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th
Counting of odd days:
1600 years have 0 odd day. 300 years have 1 odd day.
47 years = (11 leap years + 36 ordinary years)
= [(11 x 2) + (36 x 1») odd days = 58 odd days = 2 odd days.
Jan. Feb. March April May June July Aug.
31 + 28 + 31 + 30 + 31 + 30 + 31 + 15
= 227 days = (32 weeks + 3 days) = 3,
Total number of odd days = (0 + 1 + 2 + 3) odd days = 6 odd days.
Hence, the required day was 'Saturday'.

Ex. 3. What was the day of the week on 16th April, 2000 ?

Sol. 16th April, 2000 = (1999 years + Period from 1st Jan., 2000 to 16thA'
Counting of odd days:
1600 years have 0 odd day. 300 years have 1 odd day.
99 years = (24 leap years + 75 ordinary years)
= [(24 x 2) + (75 x 1)] odd days = 123 odd days
= (17 weeks + 4 days) = 4 odd days.
Jan. Feb. March April
31 + 29 + 31 + 16 = 107 days = (15 weeks + 2 days) = 2 odd,

Total number of odd days = (0 + 1 + 4 + 2) odd days = 7 odd days = 0 odd day. Hence, the required day was 'Sunday'.

Ex. 4. On what dates of Jull.2004 did Monday fall?

Sol . Let us find the day on 1st July, 2004.
2000 years have 0 odd day. 3 ordinary years have 3 odd days.
Jan. Feb. March April May June July
31 + 29 + 31 + 30 + 31 + 30 + 1
= 183 days = (26 weeks + 1 day) = 1 t .
Total number of odd days = (0 + 3 + 1) odd days = 4 odd days. '
:. 1st July 2004 was 'Thursday',-,-
Thus, 1st Monday in July 2004 _as on 5th July.
Hence, during July 2004, Monday fell on 5th, 12th, 19th and 26th. .

Ex. 5. Prove that the calendar for the year 2008 will serve for the year 20ll

Sol. In order that the calendar for the year 2003 and 2014 be the same, 1st January of both the years must be on the same day of the week.
For this, the number of odd days between 31st Dec., 2002 and 31st Dec.,2013 must be the same.
We know that an ordinary year has 1 odd day and a leap year has 2 odd During this period, there are 3 leap years, namely 2004, 2008 and 2012 and 8 ordinary years.
Total number of odd days = (6 + 8) days = 0 odd day.
Hence, the calendar for 2003 will serve for the year 2014.

Ex. 6. Prove that any date in March of a year is the same day of the week corresponding date in November that year.

We will show that the number of odd days between last day of February and last day of October is zero. .

March April May June July Aug. Sept. Oct.

31 + 30 + 31 + 30 + 31 + 31 + 30 + 31

= 241 days = 35 weeks = 0 odd day. ,Number of odd days during this period = O.

Thus, 1st March of an year will be the same day as 1st November of that year. Hence, the result follows.

28. CLOCKS

IMPORTANT FACTS

The Face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called ***minute spaces***.
A clock has two hands, the smaller one is called *the* ***hour hand*** **or** ***short hand*** while the larger one is called the ***minute hand*** **or** ***long hand..***
i) In 60 minutes, the minute hand gains 55 minutes on the hour hand.
ii) In every hour, both the hands coincide once.
iii) The hands are in the same straight line when they are coincident or opposite to each other.
iv) When the two hands are at right angles, they are 15 minute spaces apart.
v)When the hand's are in opposite directions, they are 30 minute spaces apart.
vi)Angle traced by hour hand in 12 hrs = 360°.
vii)Angle traced by minute hand in 60 min. = 360°.

Too Fast **and** ***Too Slow: If*** **a** ***watch or*** **a** ***clock indicates*** **8.15,** ***when the correct time*** **, 8 is said** ***to be*** **15** ***minutes too fast.***
On the other hand, if it indicates **7.45,** ***when the correct time is*** **8,** ***it is said to be 15 minutes too slow.***

SOLVED EXAMPLES

Ex 1:Find the angle between the hour hand and the minute hand of a clock when 3.25.

Solution:angle traced by the hour hand in 12 hours = 360°
Angle traced by it in three hours 25 min (ie) 41/12 hrs=(360*41/12*12)° =102*1/2°
angle traced by minute hand in 60 min. = 360°.
Angle traced by it in 25 min. = (360 X 25)/60= 150°
Required angle = 1500 – 102*1/2°= 47*1/2°

Ex 2: At what time between 2 and 3 o'clock will the hands of a clock be together?

Solution: At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, *i.e.* they are 10 min spaces apart.
To be together, the minute hand must gain 10 minutes over the hour hand.
Now, 55 minutes are gained by it in 60 min.

10 minutes will be gained in (60 x 10)/55 min. = 120/11 min.

The hands will coincide at 120/11 min. past 2.

Ex. 3. At what time between 4 and 5 o'clock will the hands of a clock be at right angle?

Sol: At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand, Now, when the two hands are at right angles, they are 15 min. spaces apart. So, they are at right angles in following two cases.
Case I. When minute hand is 15 min. spaces *behind the hour hand:*
In this case min. hand will have to gain (20 - 15) = 5 minute spaces. 55 min. spaces are gained by it in 60 min.

5 min spaces will be gained by it in 60*5/55 min=60/11min.

:. They are at right angles at 60/11min. past 4.

Case II. When the minute hand is 15 min. spaces *ahead* of the *hour* hand:
To be in this position, the minute hand will have to gain (20 + 15) = 35 minute spa'
55 min. spaces are gained in 60 min.

35 min spaces are gained in (60 x 35)/55 min =40/11

:. They are at right angles at 40/11 min. past 4.

Ex. 4. Find at what time between 8 and 9 o'clock will the hands of a clock being the same straight line but not together.

Sol: At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, i.e. the two hands_ are 20 min. spaces apart.
To be in the same straight line but not together they will be 30 minute spaces apart.
So, the minute hand will have to gain (30 - 20) = 10 minute spaces over the hour

hand.
55 minute spaces are gained. in 60 min.

10 minute spaces will be gained in (60 x 10)/55 min. = 120/11min.

:. The hands will be in the same straight line but not together at 120/11 min.

Ex. 5. At what time between 5 and 6 o'clock are the hands of a clock 3minapart?

. Sol. At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.
Case I. Minute hand is 3 min. spaces *behind the hour hand.*
In this case, the minute hand has to gain' (25 - 3) = 22 minute spaces. 55 min. are gained in 60 min.

22 min. are gaineg in (60*22)/55min. = 24 min.

:. The hands will be 3 min. apart at 24 min. past 5.
Case II. Minute hand is 3 min. spaces *ahead* of *the hour hand.*
In this case, the minute hand has to gain (25 + 3) = 28 minute spaces. 55 min. are gained in 60 min.

28 min. are gained in (60 x 28_)/55=346/11

The hands will be 3 min. apart at 346/11 min. past 5.

Ex 6. Tbe minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol: In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.
To be together again, the minute hand must gain 60 minutes over the hour hand. 55 min. are gained in 60 min.

60 min are gained in $\frac{60}{55}$ x 60 min =720/11 min.

But, they are together after 65 min.
Gain in 65 min =720/11-65 =5/11min.

Gain in 24 hours =(5/11 * (60*24)/65)min =440/43
The clock gains 440/43 _minutes in 24 hours.

Ex. 7. A watch which gains uniformly, is 6 min. slow at 8 o'clock in the morning Sunday and it is 6 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours = 180 hours

The watch gains (5 + 29/5) min. or 54/5 min. in 180 hrs.
Now 54/5 min. are gained in 180 hrs.

5 min. are gained in (180 x 5/54 x 5) hrs. = 83 hrs 20 min. = 3 days 11 hrs 20 min.

Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.
It will be correct at 20 min. past 7 p.m. on Wednesday.

Ex 8. A clock is set right at 6 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.
Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

356/15 hrs of this clock = 24 hours of correct clock.

89 hrs of this clock = (24 x 31556 x 89) hrs of correct clock.

= 90 hrs of correct clock.
So, the correct time is 11 p.m.

Ex. 9. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?

Sol. Time from 8 a.m. on a day 1 p.m. on the following day = 29 hours.
24 hours 10 min. of this clock = 24 hours of the correct clock.

145 /6 hrs of this clock = 24 hrs of the correct clock

29 hrs of this clock = (24 x 6/145 x 29) hrs of the correct clock

= 28 hrs 48 min. of correct clock

The correct time is 28 hrs 48 min. after 8 a.m.
This is 48 min. past 12.

29. STOCKS AND SHARES

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus(in the name of company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them **shares** of the company.

IMPORTANT FACTS AND FORMULAE

1. **Stock-capital:** The total amount needed to run the company is called the **stock-capital**
2. **Shares or stock:** The whole capital is divided into small units, called **shares** or **stock.**
   For each investment, the company issues a share-certificate, showing the value of each share and the number of shares held by a person.
   The person who subscribers in shares or stock is called a **share holder** or **stock holder.**
3. **Dividend:** The annual profit distributed among share holders is called **dividend.** Dividend is paid annually as per share or as a percentage.
4. **Face Value:** The value of a share or stock printed on the share-certificate is called its **Face Value** or **Nominal Value** or **Par Value.**
5. **Market Value:** The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be:
   (i) **At premium** or **Above par**, if its market value is more than its face value.
   (ii) **At par**, if its market value is the same as its face value.
   (iii) **At discount** or **Below par**, if its market value is less than its face value.
   Thus, if a Rs.100 stock is quoted at a premium of 16, then market value of the stock = Rs. (100+16) = Rs. 116.
   Likewise, I f a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. (100-7) = Rs. 93.
6. **Brokerage:** The broker's charge is called **brokerage.**
   (i) When stock is purchased, brokerage is added to the cost price.
   (ii) When stock is sold, brokerage is subtracted from the selling price.
   Remember:
   (i) The face value of a share always remains the same.
   (ii) The market value of a share changes form time to time.
   (iii) Dividend is always paid on the face value of a share.
   (iv) Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{face Value of 1 share}}$$

Thus, by a Rs. 100, 9% stock at 120, we mean that:

(i) Face Value (N>V) of stock = Rs. 100.
(ii) Market Value (M>V) of stock = Rs. 120.
(iii) Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.
(iv) An investment of Rs. 120 gives an annual income of Rs. 9.
(v) Rate of interest p.a = Annual income from an investment of Rs. 100.

= (9/120 * 100) % = 7 (1/2) %.

SOLVED EXAMPLES

Ex. 1. Find the cost of:

(i) Rs. 7200, 8% stock at 90;
(ii) Rs. 4500, 8.5% stock at 4 premium;
(iii) Rs. 6400, 10% stock at 15 discount.

Sol. **(i)** Cost of Rs. 100 stock = Rs. 90
Cost of Rs. 7200 stock = Rs. (90/100 * 7200) = Rs. 6480.

(ii) Cost of Rs. 100 stock = Rs. (100+4)
Cost of Rs. 4500 stock = Rs. (104/100 * 4500) = Rs. 4680

(iii) Cost of Rs. 100 stock = Rs. (100-15)
Cost of Rs. 6400 stock = Rs. (85/100 * 6400) = Rs. 5440.

Ex. 2. Find the cash required to purchase Rs. 3200, 7(1/2) % stock at 107 (brokerage (1/2) %)

Sol. Cash required to purchase Rs. 100 stock = Rs (107+(1/2)) = Rs. (215/2).
Cash required to purchase Rs. 100 stock = Rs [(215/2)*(1/100)*3200] = Rs. 3440.

Ex. 3. Find the cash realised by selling Rs. 2440, 9.5% stock at 4 discount (brokerage (1/4) %)

Sol. By selling Rs. 100 stock , cash realised = Rs. [(100-4)-(1/4)] = Rs. (383/4).
By selling Rs. 2400 stock, cash realised = Rs. [(383/4)*(1/100)*2400] = Rs 2298.

Ex. 4. Find the annual income derived from Rs. 2500, 8% stock at 106.

Sol. Income from Rs. 100 stock = Rs. 8.
Income from Rs. 2500 = Rs. [(8/1000*2500) =Rs. 200.

Ex. 5. Find the annual income derived by investing Rs. 6800 in 10% stock at 136.

Sol. By investing Rs. 136, income obtained = Rs. 10.

By investing Rs. 6800, income obtained = Rs. [(10/136)*6800] = Rs. 500.

Ex. 6. Which is better investment? 7(1/2) % stock at 105 or 6(1/2) % at 94.

Sol. Let the investment in each case be Rs. (105*94).

Case I : 7(1/2) 5 stock at 105:
On investing Rs. 105, income = Rs. (15/2).
On investing Rs. (105*94), income = Rs. [(15/2)*(1/105)*105*94] = Rs 705.

Case II : 6(1/2) % stock at 94:
On investing Rs. 94, income = Rs. (13/2).
On investing Rs. (105*94), income = Rs. [(13/2)*(1/94)*105*94] = Rs. 682.5.
Clearly, the income from 7(1/2) % stock at 105 is more.
Hence, the investment in 7(1/2) % stock at 105 is better.

Ex. 7. Find the cost of 96 shares of Rs. 10 each at (3/4) discount, brokerage being (1/4) per share.

Sol. Cost of 1 share = Rs. [(10-(3/4)) + (1/4)] = Rs. (19/2).
Cost of 96 shares = Rs. [(19/2)*96] = Rs. 912.

Ex. 8. Find the income derived from 88 shares of Rs. 25 each at 5 premium, brokerage being (1/4) per share and the rate of dividend being 7(1/2) % per annum. Also, find the rate of interest on the investment.

Sol. Cost of 1 share = Rs. [25+5+1/4)] = Rs. (121/4).
Cost of 88 shares = Rs.[(121/4)*88] = Rs. 2662.
∴ Investment made = Rs. 2662.
Face value of 88 shares = Rs. (88*25) = Rs. 2200.
Dividend on Rs. 100 = (15/2).
Dividend on Rs. 2200 = Rs. [(15/20*(1/100)*2200] = Rs. 165.
∴ Income derived = Rs. 165.
Rate of interest on investment = [(165/2662)*100] = 6.2 %.

Ex. 9. A man buys Rs. 25 shares in company which pays 9 % dividend. The money invested is such that it gives 10 % on investment. At what price did he buy the shares?

Sol. Suppose he buys each share for Rs. x.
Then, [25*(9/100)] = [x*(10/100)] or x = Rs. 22.50.
Cost of each share = Rs. 22.50.

Ex. 10. A man sells Rs.5000, 12 % stock at 156 and uinvests the proceeds parity in 8 % stock at 90 and 9 % stock at 108. He hereby increases his income by Rs. 70. How much of the proceeds were invested in each stock?

Sol. S.P of Rs. 5000 stock = Rs. [(156/100)*5000] = Rs. 7800.

Income from this stock = Rs. [(12/100)*5000] = Rs. 600.
Let investment in * % stock be x and that in 9 % stock = (7800-x).

∴ [x*(8/90)] + (7800-x) * (9/108) = (600+7)

⇔ (4x/45) + [(7800-x)/12] = 670 ⇔ 16x + 117000-15x = (670*180) ⇔ x = 3600.

∴ Money invested in 8 % stock at 90 = Rs. 3600.
Money invested in 9 % at 108 = Rs. (7800-3600) = Rs. 4200.

30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation: Let n be a positive integer. Then, factorial n, denoted by n! is defined as:

$$n! = n(n-1)(n-2)........3.2.1.$$

Examples: (i) 5! = (5x 4 x 3 x 2 x 1) = 120; (ii) 4! = (4x3x2x1) = 24 etc.
We define, 0! = 1.

Permutations: The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are: **(ab, ba, ac, bc, cb).**

Ex. 2. All permutations made with the letters a,b,c, taking all at a time are:
(abc, acb, bca, cab, cba).

Number of Permutations: Number of all permutations of n things, taken r at a time, given by:

$$^nP_r = n(n-1)(n-2).....(n-r+1) = n!/(n-r)!$$

Examples: (i) $^6p_2$ = (6x5) = 30. (ii) $^7p_3$ = (7x6x5) = 210.

Cor. Number of all permutations of n things, taken all at a time = n!

An Important Result: If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1+p_2+.......p_r) = n$.

Then, number of permutations of these n objects is:

$$n! \ / \ (p_1!).p_2!)......(p_r!)$$

Combinations: Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.
Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c, taking two at a time are **ab, bc, ca.**

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is **abc.**

Ex. 4. Various groups of 2 out of four presons A, B, C, D are:

AB, AC, AD, BC, BD, CD.

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations: The number of all combination of n things, taken r at a time is:

$$^{n}C_{r} = n! / (r!)(n-r)! = n(n-1)(n-2).....\text{to } r \text{ factors} / r!$$

Note that: $^{n}c_{r} = 1$ and $^{n}c_{0} = 1$.

An Important Result: $^{n}c_{r} = {}^{n}c_{(n-r)}$.

Example: (i) $^{11}c_{4} = (11x10x9x8)/(4x3x2x1) = 330$.

(ii) $^{16}c_{13} = {}^{16}c_{(16-13)} = 16x15x14/3! = 16x15x14/3x2x1 = 560$.

SOLVED EXAMPLES

Ex. 1. Evaluate: 30!/28!

Sol. We have, 30!/28! = 30x29x(28!)/28! = (30x29) = 870.

Ex. 2. Find the value of (i) $^{60}p_{3}$ (ii) $^{4}p_{4}$

Sol. (i) $^{60}p_{3} = 60!/(60-3)! = 60!/57! = 60x59x58x(57!)/57! = (60x59x58) = 205320$.

(ii) $^{4}p_{4} = 4! = (4x3x2x1) = 24$.

Ex. 3. Find the vale of (i) $^{10}c_{3}$ (ii) $^{100}c_{98}$ (iii) $^{50}c_{50}$

Sol. (i) $^{10}c_{3} = 10x9x8/3! = 120$.

(ii) $^{100}c_{98} = {}^{100}c_{(100-98)} = 100x99/2! = 4950$.

(iii) $^{50}c_{50} = 1$.   [$^{n}c_{n} = 1$]

Ex. 4. How many words can be formed by using all letters of the word "BIHAR"

Sol. The word BIHAR contains 5 different letters.

Required number of words = $^{5}p_{5}$ = 5! = (5x4x3x2x1) = 120.

Ex. 5. How many words can be formed by using all letters of the word 'DAUGHTER' so that the vowels always come together?

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.
Then, the letters to be arranged are DGNTR (AUE).

Then 6 letters to be arranged in $^{6}p_{6}$ = 6! = 720 ways.

The vowels in the group (AUE) may be arranged in 3! = 6 ways.

Required number of words = (720x6) = 4320.

Ex. 6. How many words can be formed from the letters of the word 'EXTRA' so that the vowels are never together?

Sol. The given word contains 5 different letters.
Taking the vowels EA together, we treat them as one letter.
Then, the letters to be arranged are XTR (EA).
These letters can be arranged in 4! = 24 ways.
The vowels EA may be arranged amongst themselves in 2! = 2 ways.
Number of words, each having vowels together = (24x2) = 48 ways.
Total number of words formed by using all the letters of the given words
= 5! = (5x4x3x2x1) = 120.

Number of words, each having vowels never together = (120-48) = 72.

Ex. 7. How many words can be formed from the letters of the word 'DIRECTOR' So that the vowels are always together?

Sol. In the given word, we treat the vowels IEO as one letter.
Thus, we have DRCTR (IEO).
This group has 6 letters of which R occurs 2 times and others are different.
Number of ways of arranging these letters = 6!/2! = 360.
Now 3 vowels can be arranged among themselves in 3! = 6 ways.
Required number of ways = (360x6) = 2160.

Ex. 8. In how many ways can a cricket eleven be chosen out of a batch of 15 players ?

Sol. Required number of ways = $^{15}c_{11} = {}^{15}c_{(15-11)} = {}^{11}c_4$

$= 15x14x13x12/4x3x2x1 = 1365.$

Ex. 9. In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

Sol. (3 men out 6) and (2 ladies out of 5) are to be chosen.

Required number of ways = $({}^6c_3 x {}^5c_2) = [6x5x4/3x2x1]$ x $[5x4/2x1] = 200.$

31. PROBABILITY
IMPORTANT FACTS AND FORMULA

1.Experiment :An operation which can produce some well-defined outcome is called an experiment

2.Random experiment: An experiment in which all possible outcome are known and the exact out put cannot be predicted in advance is called an random experiment
Eg of performing random experiment:
(i)rolling an unbiased dice
(ii)tossing a fair coin
(iii)drawing a card from a pack of well shuffled card
(iv)picking up a ball of certain color from a bag containing ball of different colors

Details:
(i)when we throw a coin. Then either a **head(h)** or a **tail (t)** appears.
(ii)a dice is a solid cube, having 6 faces ,marked 1,2,3,4,5,6 respectively when we throw a die , the outcome is the number that appear on its top face .
(iii)a pack of cards has 52 cards it has 13 cards of each suit ,namely spades, clubs ,hearts and diamonds
Cards of spades and clubs are black cards
Cards of hearts and diamonds are red cards
There are 4 honors of each suit
These are **aces ,king ,queen and jack**
These are called face cards

3.S**ample space :W**hen we perform an experiment ,then the set S of all possible outcome is called the sample space
eg of sample space:
(i)in tossing a coin ,s={h,t}
(ii)if two coin are tossed ,then s={hh,tt,ht,th}.
(iii)in rolling a die we have,s={1,2,3,4,5,6}.

4.event:Any subset of a sample space.
5.Probability of occurrence of an event.
let S be the sample space and E be the event .
then,$E\subseteq S$.
$P(E)=n(E)/n(S)$.

6.Results on Probability:
(i)$P(S) = 1$ (ii)$0\leq P(E)\leq 1$ (iii)$P(\phi)=0$
(iv)For any event a and b, we have:
$P(a\cup b)=P(a)+P(b)-P(a\cup b)$

(v)If $\overline{A}$ denotes (not-a),then P($\overline{A}$)=1-P(A).

SOLVED EXAMPLES

Ex 1. In a throw of a coin ,find the probability of getting a head.
sol. Here s={H,T} and E={H}.
P(E)=n(E)/n(S)=1/2

Ex2.Two unbiased coin are tossed .what is the probability of getting atmost one head?
sol.Here S={HH,HT,TH,TT}
Let Ee=event of getting one head
E={TT,HT,TH}
P(E)=n(E)/n(S)=3/4

Ex3.An unbiased die is tossed .find the probability of getting a multiple of 3
sol. Here S={1,2,3,4,5,6}
Let E be the event of getting the multiple of 3
then ,E={3,6}
P(E)=n(E)/n(S)=2/6=1/3

E**x4.** **I**n a simultaneous throw of pair of dice .find the probability of getting the total more than 7
sol. Here n(S)=(6*6)=36
let E=event of getting a total more than 7

={(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)}
P(E)=n(E)/n(S)=15/36=5/12.

Ex5. A bag contains 6 white and 4 black balls .2 balls are drawn at random. find the probability that they are of same colour.
Sol .let S be the sample space
Then n(S)=no of ways of drawing 2 balls out of (6+4)=10c2=(10*9)/(2*1)=45
Let E=event of getting both balls of same colour
Then n(E)=no of ways(2 balls out of six) or(2 balls out of 4)
=(^{6}c2+^{4}c2)=(6*5)/(2*1)+(4*3)/(2*1)=15+6=21
P(E)=n(E)/n(S)=21/45=7/15

Ex6.Two dice are thrown together .What is the probability that the sum of the number on the two faces is divided by 4 or 6
sol. Clearly n(S)=6*6=36
Let E be the event that the sum of the numbers on the two faces is divided by 4 or 6.Then

E={(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(5,1),(5,3),(6,2),
(6,6)}
n(E)=14.
Hence p(e)=n(e)/n(s)=14/36=7/18

Ex7.Two cards are drawn at random from a pack of 52 cards.what is the probability that either both are black or both are queen?
sol. We have n(s)=52c2=(52*51)/(2*1)=1326.
Let A=event of getting both black cards
B=event of getting both queens
$A \cap B$=event of getting queen of black cards
n(A)=^{26}c2=(26*25)/(2*1)=325,
n(B)=^{4}c2=(4*3)/(2*1)=6 and
$n(A \cap B)$=2c2=1
P(A)=n(A)/n(S)=325/1326;
P(B)=n(B)/n(S)=6/1326 and
$P(A \cap B)=n(A \cap B)/n(S)$=1/1326
$P(A \cup B)=P(A)+P(B)-P(A \cap B)$=(325+6-1/1326)=330/1326=55/221

32. TRUE DISCOUNT

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that:
Sum due = Rs. 156 due 4 years hence;
Present Worth (P.W.) = Rs. 100;
True Discount (T.D.) = Rs. (156 - 100) = Rs. 56
(Sum due) - (P.W.).
We define : T.D. = Interest on P.W.
Amount = (P.W.) + (T.D.).
Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

1. P.W.=[100 x Amount /100 + (R x T)
   =100 x T.D./ RxT
2. T.D.=[(P.W.) x R x T /100]
   = [Amount x RxT/100 + (R x T)]
3. (S.I.)*(T.D.) /(S.I.)-(T.D.)
4. (S.I.) - (T.D.) - S.I. on T.D.

5. When the sum is put at compound interest, then
P.W. = Amount/[1 +R/100]^T

SOLVED EXAMPLES

Ex. 1. Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.
Sol.

P.W=100 x Amount /[100 + (R x T)]

=Rs.100 x 930/100+ (8x3)
= (100x930)/124
= Rs. 750,
T.D. = (Amount) - (P.W.) = Rs. (930 - 750) = Rs. 180.

Ex. 2. The true discount on a bill due 9 months hence at 12% per annum is Rs. Find the amount of the bill and its present worth.

Sol. Let amount be Rs. x. Then,
x*R*T/100 + (R x T)
=T.D.
=>x * 12*3/ 4/[100+[12*3/4]]
=540
x= 540x109 = Rs.6540
Amount - Rs. 6540. P.W. = Rs. (6540 - 540) - Rs. 6000.

Ex. 3. The true discount on a certain sum of money due 3 years hence is Rb. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.

Sol. T.D. = Rs. 250 and S.I. = Rs. 375.
Sum due =S.I. xT.D./ S.I. -T.D.

=375x250/375- 250
=Rs.750.
Rate=[100*375/750*3]%=16 2/3%

Ex. 4. The difference between the simple interest and true discount on a certain sum
of money for 6 months at 12—% per annum is Rs. 25. Find the

sum.

Sol. Let the sum be Rs. x. Then,
T.D. = (x*25/2*1/2)/(100+(25/2*1/2))=x*25/4*4/425=x/17
S.I=x*25/2*1/2*1/100=x/16
x/16-x/17=25
=>17x-16x=25*16*17
=>x=6800
Hence, sum due = Rs. 6800.

Ex. 5. A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement
ins Rb. 40. What is the amount of the bill, if the money be worth 12-z% ?
Sol. Let the sum be Rs. x. Then,
[x/2+(x/2*100)/100+(25/2*2)]-[(x*100)/(100+25/2*1]

=40
=>x/2+2x/5-8x/9=40
=>x=3600
Amount of the bill - Rs. 3600.

33. BANKER'S DISCOUNT

IMPORTANT CONCEPTS

Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.
The date exactly after 5 months is called nominally due date. Three days (known as grace days) are added to it to get a date, known as legally due date.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (i.e., Rs. 10,000 in this case) for the period from the date on which the bill was discounted (i.e., paid by the banker) and the legally due date. This amount is known as Banker's Discount (B.D.) Thus, B.D. is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.

Banker's Gain (B.G.) = (B.D.) - (T.D.) for the unexpired time.
Note : When the date of the bill is not given, grace days are not to be added.

IMPORTANT FORMULAE

1. B.D. = S.I. on bill for unexpired time.
2. B.G. = (B.D.)-(T.D.) = S.I. on T.D. $=(T.D)^2/P.W$.

3. T.D. = √(P.W.xB.G.)

4. B.D. =[(Amount *Rate *Time)/100]
5. Amount=[(B.D. x T.D.)/(B.D.-T.D.)]
6. T.D.= [(Amount x Rate x Time)/(100+(Rate*Time))]
7. T.D.=[(B.G. x 100)/(Rate x Time)]

SOLVED EXAMPLES

Ex. 1. A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.
Sol.
Face value of the bill = Rs. 6000.
Date on which the bill was drawn = July 14 at 5 months. Nominally due date = December 14.
Legally due date = December 17.
Date on which the bill was discounted = October 5.
Unexpired time : Oct. Nov. Dec.
26 + 30 + 17 = 73 days =1/ 5Years

B.D. = S.I. on Rs. 6000 for 1/5 year
= Rs. (6000 x 10 x1/5 x1/100)= Rs. 120.

T.D. = Rs.[(6000 x 10 x1/5)/(100+(10*1/5))]
=Rs.(12000/102)=Rs. 117.64.

B.G. = (B.D.) - (T.D.) = Rs. (120 - 117.64) = Rs. 2.36.

Money received by the holder of the bill = Rs. (6000 - 120)
= Rs. 5880.

Ex. 2. If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?

Sol. B.G. = S.I. on T.D.
= Rs.(120 x 15 x 1/2 x 1/100)
= Rs. 9.
(B.D.) - (T.D.) = Rs. 9.
B.D. = Rs. (120 + 9) = Rs. 129.

Ex. 3. The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.
Sol.
S.I. on Rs. 1800 = T.D. on Rs. 1872.
P.W. of Rs. 1872 is Rs. 1800.
Rs. 72 is S.I. on Rs. 1800 at 12%.
Time =[(100 x 72)/ (12x1800)]year
1/3year = 4 months.

Ex. 4. The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.
Sol.
Sum =[(B.D.*T.D.)/(B.D.-T.D.)]
= Rs.[(120x110)/(120-110)]
= Rs. 1320.
Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.
Rate =[(100 x120)/(1320 x 2/3)%
= 13 7/11%.

Ex. 5. The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.
Sol. T.D. =√(P.W.*B.G)
B.G. =(T.D.)2/ P.W.
= Rs.[(110x110)/ 1100]

= Rs. 11.
B.D.= (T.D. + B.G.) = Rs. (110 + 11) = Rs. 121.

Ex. 6. The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.
Sol.
Sum = [(B.D.xT.D.)/ (B.D.-T.D.)]
= [(B.D.xT.D.)/B.G.]
T.D./B.G. = Sum/ B.D.
=1650/165
=10/1
Thus, if B.G. is Re 1, T.D. = Rs. 10.
If B.D.is Rs. ll, T.D.=Rs. 10.
If B.D. is Rs. 165, T.D. = Rs. [(10/11)xl65]
=Rs.150
And, B.G. = Rs. (165 - 150) = Rs, 15.

Ex. 7. What rate percent does a man get for his money when in discounting a bill due 10 months hence, he deducts 10% of the amount of the bill?
Solution: Let amount of the bill = Rs.100
Money deducted =Rs.10
Money received by the holder of the bill = Rs.100-10 = Rs.90
SI on Rs.90 for 10 months = Rs.10
Rate =[(100*10)/(90*10/12)%=13 1/3%

34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1.We already know that:

In a rt.angled ΔOAB, where $\angle BOA = \theta$,

i)$\sin\theta$ = Perpendicular/Hypotenuse = AB/OB;

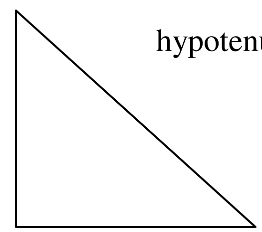
ii)$\cos\theta$ = Base/Hypotenuse = OA/OB;

iii)$\tan\theta$ = Perpendicular/Base = AB/OA;

iv)$\operatorname{cosec}\theta = 1/\sin\theta$ = OB/AB;

v)$\sec\theta = 1/\cos\theta$ = OB/OA;

vi)$\cot\theta = 1/\tan\theta$ = OA/AB.



2. **Trigonometrical identities:**

i)$\sin^2\theta + \cos^2\theta = 1$.

ii)$1+\tan^2\theta = \sec^2\theta$

iii)$1+\cot^2\theta = \operatorname{cosec}^2\theta$

3. **Values of T-ratios:-**

θ	0	30°	45°	60°	90°
Sin θ	0	½	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	½	0
Tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined

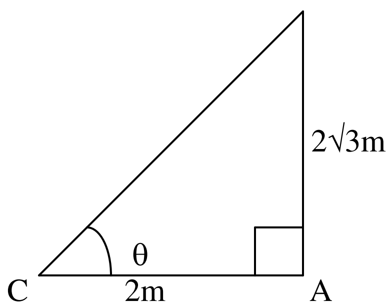
4. **Angle of Elevation:** Suppose a man from a point O looks up an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

$\therefore$ Angle of elevation of P from O = $\angle AOP$.

5. **Angle of Depression**: Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.

SOLVED EXAMPLES

Ex.1.If the height of a pole is 2√3 metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.



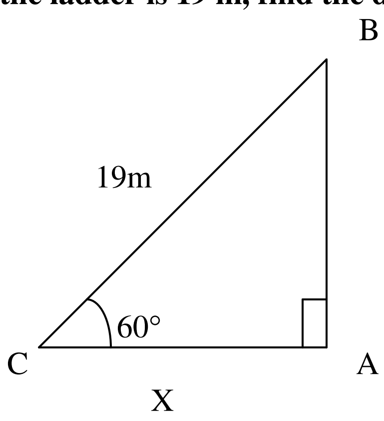
Sol. Let AB be the pole and AC be its shadow.
Let angle of elevation, $\angle ACB=\theta$.
Then, AB = $2\sqrt{3}$ m AC = 2 m.

$\tan \theta = AB/AC = 2\sqrt{3}/2 = \sqrt{3} \Rightarrow \theta = 60°$

So, the angle of elevation is 60°

Ex.2. A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.

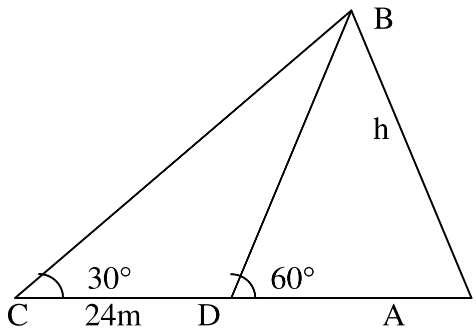
2)



Sol. Let AB be the wall and BC be the ladder.
Then, $\angle ACB = 60°$ and BC = 19 m.
Let AC = x metres
$AC/BC = \cos 60° \Rightarrow x/19 = ½ \Rightarrow x=19/2 = 9.5$

$\therefore$ Distance of the foot of the ladder from the wall = 9.5 m

Ex.3. The angle of elevation of the top of a tower at a point on the ground is 30°. On walking 24 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.



Sol. Let AB be the tower and C and D be the points of observation. Then,

$AB/AD = \tan 60° = \sqrt{3}$ => $AD = AB/\sqrt{3} = h/\sqrt{3}$

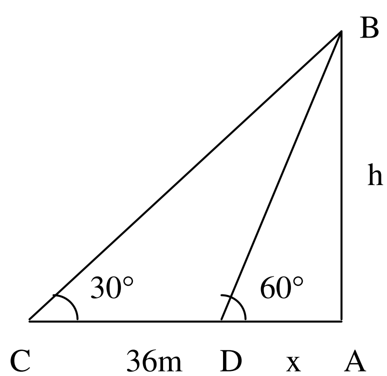
$AB/AC = \tan 30° = 1/\sqrt{3}$ $AC = AB \times \sqrt{3} = h\sqrt{3}$

$CD = (AC - AD) = (h\sqrt{3} - h/\sqrt{3})$

$h\sqrt{3} - h/\sqrt{3} = 24$ => $h = 12\sqrt{3} = (12 \times 1.73) = 20.76$

Hence, the height of the tower is 20.76 m.

Ex.4. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 36 m from the bank, he finds the angle to be 30°. Find the breadth of the river.



Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$\angle ACB=60°$, $\angle ADB=30°$ and CD=36 m.

Let AB=h metres and AC=x metres.

Then, AD=(36+x)metres.

$AB/AD=\tan 30°=1/\sqrt{3}$ => $h/(36+x)=1/\sqrt{3}$

$h=(36+x)/\sqrt{3}$ .....(1)

$AB/AC=\tan 60°=\sqrt{3}$ => $h/x=\sqrt{3}$

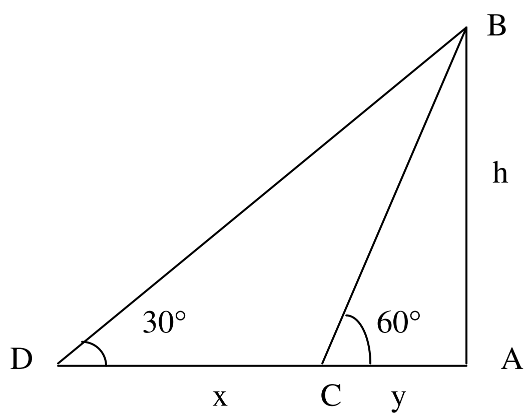
$h=\sqrt{3}x$ .....(2)

From (i) and (ii), we get:

$(36+x)/\sqrt{3}=\sqrt{3}x$ => $x=18$ m.

So, the breadth of the river = 18 m.

Ex.5. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.



Sol. Let AB be the tower and C and D be the two positions of the boat.

Let AB=h, CD=x and AD=y.

$h/y=\tan 60°=\sqrt{3}$ => $y=h/\sqrt{3}$

$h/(x+y)=\tan 30°$ = $1/\sqrt{3}$ => $x+y=\sqrt{3}h$

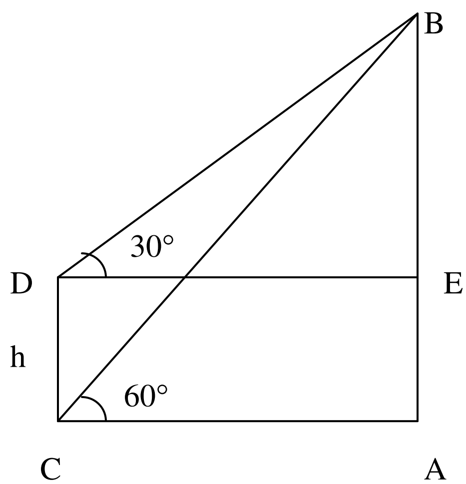
$x=(x+y)-y=(\sqrt{3}h-h/\sqrt{3})=2h/\sqrt{3}$

Now, $2h/\sqrt{3}$ is covered in 10 min.

$h/\sqrt{3}$ will be covered in $(10\times(\sqrt{3}/2h)\times(h/\sqrt{3}))=5$ min

Hence, required time = 5 minutes.

Ex 6. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.



Sol. Let AB and CD be the two temples and AC be the river.
Then, AB = 54 m.
Let AC = x metres and CD=h metres.

$\angle ACB=60°$, $\angle EDB=30°$
$AB/AC=\tan 60°=\sqrt{3}$
$AC=AB/\sqrt{3}=54/\sqrt{3}=(54/\sqrt{3}\times\sqrt{3}/\sqrt{3})=18m$
$DE=AC=18\sqrt{3}$
$BE/DE=\tan 30°=1/\sqrt{3}$
$BE=(18\sqrt{3}\times 1/\sqrt{3})=18$ m
$CD=AE=AB-BE=(54-18)$ m = 36 m.
So, Width of the river = AC = $18\sqrt{3}$ m=18×1.73 m=31.14m
Height of the other temple = CD= 18 m.

36. TABULATION

This section comprises of questions in which certain data regarding common disciplines as production over a period of a few years: imports, exports, incomes of employees in a factory, students applying for and qualifying a certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given questions on the basis of comparative analysis of the data.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the *tabular form.* In order to avoid some heads again and again, tables are made consisting of horizontal lines called *rows* and vertical lines called *columns* with distinctive heads, known as *captions.* Units of measurements are given with the captions.

SOLVED EXAMPLES

The following table gives the sales of batteries manufactured by a *company* lit *the years.* Study *the table and answer the questions that follow:*

(S.B.I.P.O. 1998)

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEARS (NUMBERS _N THOUSANDS)

TYPES OF BATTERIES

Year	4AH	7AH	32AH	35AH	55AH	T0TAL
1992	75	144	114	102	108	543
1993	90	126	102	84	426	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery ?
(a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
2. What is the difference in the number of 35AH batteries sold in 1993 and 1997 ?
(a) 24000 (b) 28000 (c) 35000 (d) 39000 (e) 42000
3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year:
(a) 1994 . (b) 1995 (c) 1996 (d) 1997 (e) 1998
4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997 ?
(8) 4AH (b) 7 AH (c) 32AH (d) 35AH (e) 55AH
5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992 ?
(a) 28% (b) 31% (c) 33% (d)34% (e)37%

Sol. 1. *(c)* : The total sales (in thousands) of all the seven years for various batteries are:
For 4AH = 75 + 90 + 96 + 105 + 90 + 105 + 115 = 676
For 7AH = 144 + 126 + 114 + 90 + 75 + 60 + 85 = 694
For 32AH = 114 + 102 + 75 + 150 + 135 + 165 + 160 = 901
For 35 AH= 102 + 84 + 105 + 90 + 75 + 45 + 100 = 601
For 55 AH= 108 + 126 + 135 + 75 + 90 + 120 + 145 = 799.
Clearly, sales are maximum in case of *32AH* batteries.
2. *(d* : Required difference = [(84 - 45) x 1000] = 39000.
3. *(d)* : The percentages of sales of 4AH batteries to the total sales in different years are:

For 1992 =(75*100/543)%=13.81%

For 1993=(90*100)/528%=17.05%

For 1994=(96*100/465)%=19.35%

For 1995=(105*100/495)%=20.59%

For 1996=(96*100/465)%=19.35%

For 1997=(105*100/495)%=21.21%

For 1998=(115*100/605)%=19.01%

Clearly, the percentage is maximum in 1997.

4. (b) : From the table it is clear that the sales of 7AH batteries have

been decreasing continuously from 1992 to 1997.

5. (d) : Required Percentage =(145-108)/108)*100 %=34.26%=34%.

Ex 2: Study the following table carefully and answer these questions:

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEAR

	1997		1998		1999		2000		2001	
	App.	Qual.	App.	Qal.	App.	Qual.	App.	Qual.	App.	Qual
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

1. Combining the states P and Q, together in 1998, what is the percentage of the candidates qualified to that of the canditates appeared?
(8) 10.87% *(b)* 11.49% *(c)* 12.35% (d) 12.54% *(e)* 13.50%

2.The percentage of the total number of qualified candidates to the total number appeared candidates among all the five states in 1999 is :
(a) 11.49% *(b)* 11.84% *(c)*- 12.21% (d) 12.57% *(e)* 12.7a1

3. What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together?
(a) 12.36% *(b)* 12.16% *(c)* 11.47% (d) 11.15%(e)None of these

4. What is the average of candidates who appeared from State Q during the given yeas?
(8) 8700 *(b)* 8760 *(c)* 8810 (d) 8920 *(e)* 8990

5 . In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates?
(8) 1997 *(b)* 1998 *(c)* 1999 (d) 2000 (e) 2001

6. Total number of candidates qualified from all the states together in 1997

is approximately what percentage of the total number of candidates qualified from all the states together in 1998 ?

(8) 72% *(b)* 77% *(c)* 80% (d) 83% *(e)* 86%

Sol.1.(c)Required Percentage=(1020+1240) *100%=(2260*100)/18300%

(8800+9500)

=12.35%

Required Percentage= (850+920+890+980+1350) *100%

(7400+8450+7800+8700+9800)

=(4990*100)/42150%

=11.84%

(e) : Required Percentage=(84-+1050+920+980+1020)/(7500+9200+8450+9200+8800)*100%

=(4810*100)/43150* %

=11.15%

4. (e) Required average =(8100+9500+8700+9700+8950)/5

=44950/5

=8990

5. (e) : The percentages of candidates qualified to candidates appeared from State P during different years are:

For 1997= 780 * 100% =12.19%

6400

for 1998 = 1020*100 %=11.59%

8800

For 1999 = 890*100 %=11.41%;

7800

For 2000 = 1010* 100 % = 11.54%.

8 750

For 2001=1250*100 %= 12.82%

9750

:. Maximum percentage is for the year 2001.

6. *(c)* : Required Percentage $= \frac{(720 + 840 + 780 + 950 + 870)}{980+1050+1020+1240+940} \times 100$

$=80\%$

Ex. 3. *The following table gives the percentage of marks obtained by* seven *students* in six , *different subjects* in an *examination. Study the table* and answer *the questions* based on *it. The numbers* in *the brackets give the maximum marks* in *each subject.*
(Bank P.O. 2003)

\ (Max. marks) Student	Maths (160)	Chemistry (130)	Physics (120)	Geography (100)	History (60)	Computer Science (40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tharun	65	35	50	77	80	80

1. What was the aggregate of marks obtained by Sajal in all the six subjects?
(a) 409 *(b)* 419 *(c)* 429 (d) 439 *(e)* 449
2. What is the overall percentage of Thrun?
(a) 52.5% *(b)* 55% *(c)* 60% (d) 63% *(e)* 64.5%
3. What are the average marks obtained by all the seven students in Physics? (rounded off to two digits after decimal)
(a) 77.26 *(b)* 89.14 *(c)* 91.37 (d) 96.11 *(e)* 103.21
4. The number of students who obtained 60% and above marks in all the subjects is :
(a) 1 *(b)* 2 *(c)* 3 (d) None *(e)* None of these
6. In which subject is the overall percentage the best?
(a) History *(b)* Maths *(c)* Physics (d) Chemistry *(e)* Geography

Sol. 1.. *(e)* : Aggregate marks obtained by Sajal

$= [(90\% \text{ of } 150) + (60\% \text{ of } 130) + (70\% \text{ of } 120) + (70\% \text{ of } 100) + (90\% \text{ of } 60) + (70\% \text{ of } 40)] = 135 + 78 + 84 + 70 + 54 + 28 = 449.$

2. *(c)* : Aggregate marks obtained by Tarun .

$= [(65\% \text{ of } 150) + (35\% \text{ of } 130) + (50\% \text{ of } 120) + (77\% \text{ of } 100) + (80\% \text{ of } 60) + (80\% \text{ of } 40)] = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.$

Total maximum marks (of all the six subjects)

$= (150 + 130 + 120 + 100 + 60 + 40) = 600.$

Overall percentage of Tarun $= \frac{360 \times 100}{600}\,\% = 60\%.$

3. *(b)* : Average marks obtained in Physics by all the seven students

$$= \frac{1}{7}[(90\% \text{ of } 120) + (80\% \text{ of } 120) + (70\% \text{ of } 120) + (80\% \text{ of } 120) + (85\% \text{ of } 120) + (65\% \text{ of } 120) + (50\% \text{ of } 120)]$$

$$= \frac{1}{7}[(90 + 80 + 70 + 80 + 85 + 65 + 50)\% \text{ of } 120]$$

$$= \frac{1}{7}[520\% \text{ of } 120] = 89.14.$$

4. *(b)* : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.

6. *(b)* : We shall find the overall percentage (for all the seven students) with respect to each subject.

The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.

Therefore, overall percentage for:

(i) Maths $= [\frac{1}{7}(90+100+90+80+80+70+65)]\%$

$= [\frac{1}{7}(575)]\% = 82.14\%.$

(ii) Chemistry $= [\frac{1}{7}(50 + 80 + 60 + 65 + 65 + 75 + 35)]\%$

$= [\frac{1}{7}(430)]\% = 61.43\%.$

(iii) Physics = $[\frac{1}{7}(90 + 80 + 70 + 80 + 85 + 65 + 50)]\%$

$=[\frac{1}{7}(520)]\% = 74.29\%.$

(iv) Geography = $[\frac{1}{7}(60 + 40 + 70 + 80 + 95 + 85 + 77)]\%$

$= [\frac{1}{7}(507)\}\_ = 72.43\%.$

(v) History = $[\frac{1}{7}(70 + 80 + 90 + 60 + 50 + 40 + 80)]\%$

$=\frac{1}{7}[(470)]\% = 67.14\%.$

(vi) Computer Science = $[1/7\ (80 + 70 + 70 + 60 + 90 + 60 + 80)]\%$

$= [\frac{1}{7}(510)]\% = 72.86\%.$

Clearly; this. percentage is highest for Maths.

ex.4. *Study the following table carefully and answer tbe questions given below:*(Bank P.O. 2001)

CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION

Marks out Of 50 Subject	40 and above	30 and Above	20 and above	10 and above	0 and above
physics	9	32	80	92	100
chemistry	4	,21	66	,. 81,	100
(aggregate Average)	7	27	73	87	100

1. The number of students scoring less than 40% marks in aggregate is :
(a) 13 *(b)* 19 *(c)* 20 *(d)* 27 *(e)* 34

2. If at least 60% marks in Physics are required for pursuing higher studies in Physics,how many students will be eligible to pursue higher studies in Physics?
(a) 27 *(b)* 32 *(c)* 34 *(d)41* *(e)* 68

3. What is the difference between the number of students passed with 30 as

cut-off marks in Chemistry and those passed with :JUas cut-off marks in aggregate?

(a) 3 *(b)* 4 *(c)* 5 *(d)* 6 *(e)* 7

4. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately:

(a) 21% *(b)* 27% *(c)* 29% (d) 31% (e) 34%

5. If it is known that at least 23 students were eligible for a Symposium on Chemistry the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range:

(a) 40-50 *(b)* 30-40 *(c)* 20-30 (d) Below 20

Sol. 1. (d) : We have 40% of 50 $=\left(\frac{40}{100} \times 50\right) = 20$.

:. Required number = Number of students scoring less than 20 marks in aggregate

= 100 - number of students scoring 20 and above marks in aggregate = 100 - 73 = 27.

2. *(b)* : We have 60% of 50 $=\left(\frac{60}{100} \times 50\right) = 30$.

:. Required number = Number of students scoring 30 and above mark in Physics = 32.

3. (d) : Required difference = (Number of students scoring 30 and above in mark in Chemistry) (Number of students scoring 30 and above marks in aggregate) = 27 - 21 = 6.

4. *(c)* : Number of students getting at least 60% marks in Chemistry = Number of students getting 30 and above marks in Chemistry = 21.
Number of students getting at least 40% marks in aggregate
= Number of students getting 20 and above marks in aggregate = 73.
:. Required Percentage $= \left(\frac{21}{73} \times 100\right)\% = 28.77\% \approx 29\%$.

6. *(c)* : Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

37.BAR GRAPHS

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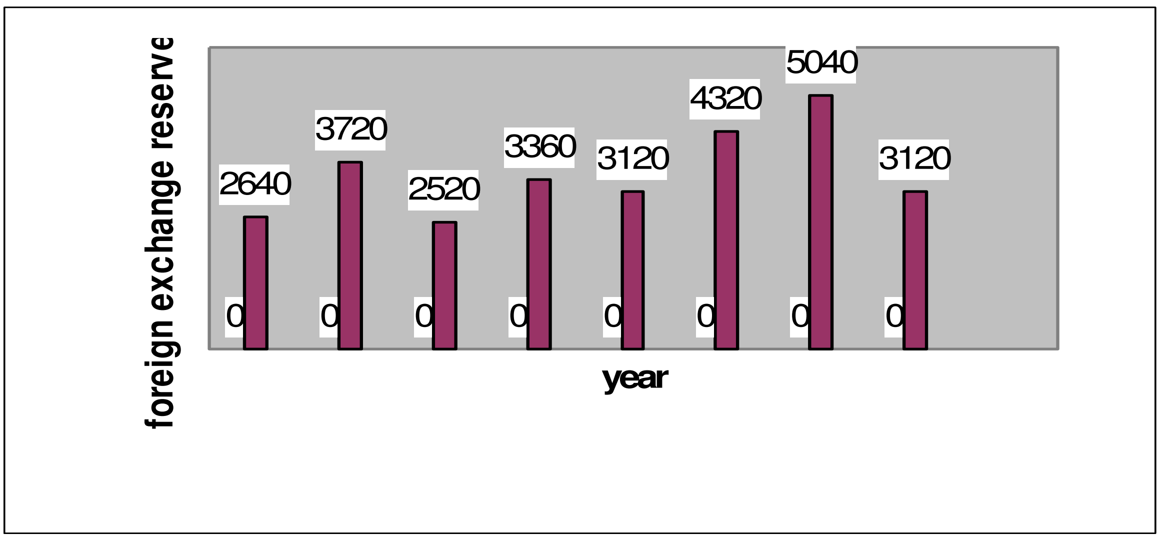
this section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale.one of the parameters is plotted on the horizontal axis and the other on the vertical axis . the candidate is required to understand the given information and thereafetr answer the given questions on the basis of data analysis.

1.The bar graph given below shows the foreign exchange reserves of a country (in million us$)
from 1991-92 to 1998-99 .answer the questions based
on this graph.

FOREIGN EXCHANGE RESERVES OF A COUNTRY
(IN MILLION US $)



1.The foreign exchange reserves in 1997-98 was how ,any times that in 1994-95?
(a)0.7 (b) 1.2 (c) 1.4 (d) 1.5 (e) 1.8
2.what was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?
(a)100 (b)150 (c)200 (d)620 (e)2520

3.for which year,the percent increase of foreign exchange reserves over the previous year,is the highest?
(a)1992-93 (b)1993-94 (c)1994-95 (d)1996-97 (e)1997-98

4.the foreign exchange reserves in 1996-97 werw approximately what percent of the average foreign exchange reserves over the period under review?

(a)95% (b)110% (c)115% (d)125% (e)140%

5.the ratio of the number of years,in which the foreign exchange reserves are above the average reserves ,to those in which the reserves are below the average reserves is :
(a)2:6 (b)3:4 (c)3:5 (d)4:4 (e)5:3

Solutions
1 (d) : required ratio = 5040/3360 = 1.5
2 (a) : foreign exchange reserve in 1997-98=5040 million us $
foreign exchange reserves in 1993-94=2520 million us$
therefore increase=(5040-2520)=2520 million us $
therefore percentage increase=((2520/2520)*100)%=100%
3(a): there is an increase in foreign exchange reserves during the years 1992-93,1994-951996-97,1997-98 as compared to previous year (as shown by bar graph)
the percentage increase in reserves during these years compared to previous year are
(1) for 1992-93 =[(3720-2640)/2640*100]% =40.91%
(2) for 1994-95=[(3360-2520)/2520*100]%=33.33%
(3) for 1996-97=[(4320-3120)/3120*100]%=38.46%
(4) for 1997-98=[(5040-4320)/4320*100]%=16.67%

Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period
= [_x (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)] million US $

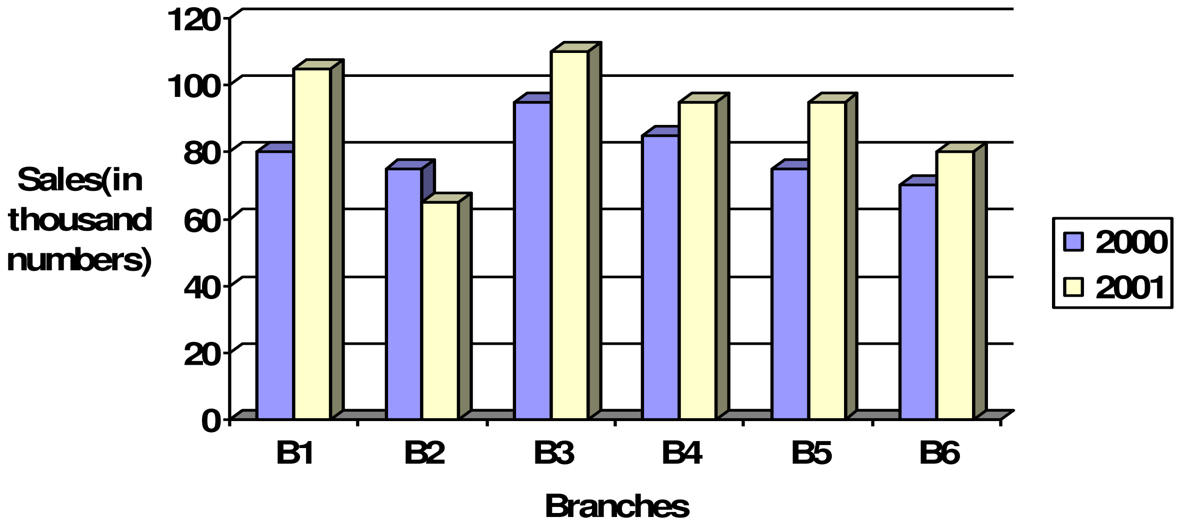
= 3480 million US $.
Foreign exchange reserves in 1996-97 = 4320 million US $. . .
Required Percentage = x 100 % = 124.14% .. 125%.
3480 .

5. *(c)* : Average foreign exchange reserves over the given period = 3480 million US $.

The country had reserves above 3480 million US $ during the years 199293, 1996-97 and 1997-98 *i.e.,* for 3 years and below 3480 million US $ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 *i.e.,* for 5 years.
Hence, required ratio = 3 : 5.

Ex. 2. The bar-graph provided on next page gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph:

Sales of books (in thousand numbers) from six branches-B1,B2,B3,B4,B5 and B6 of a publishing company in 2000 and 2001



1.total sales of branches b1,b3 and b5 together for both the years (in thousand numbers) is:
(a)250 (b) 310 (c) 435 (d)560 (e)585

2.total sales of branch b6 for both the years is what percent of the total sales of branch b3 for both the years?
(a) 68.54% (b)71.11% (c) 73.17% (d)75.55% (e)77.26%

3.what is the average sale of all the branches (in thousand numbers) for the year 2000?
(a)73 (b)80 (c)83 (d)88 (e)96

4.what is the ratio of the total sales of branch b2 for both years to the total sales of branch b4 for both years ?
(a)2:3 (b)3:5 (c)4:5 (d)5:7 (e)7:9

5.what percent of the average sales of branchesn b1,b2 and b3 in 2001 is the average sales of branches b1,b3 and b6 in 2000?
(a)75%(b)77.5% (c)82.5% (d)85% (e)87.5%

SOLN…………………..
1.(d) total sales of branches B1,B3 and B5 for both the years (in thousand numbers)=(80+105)+(95+110)+(75+95)=560

2(c) required percentage=[(70+80)/(95+110)*100]%=(150/205*100)%=73.17%

3(b)average sales of all the six branches (in thousand numbers) for the year 2000=1/6*(80+75+95+85+75+70)=80

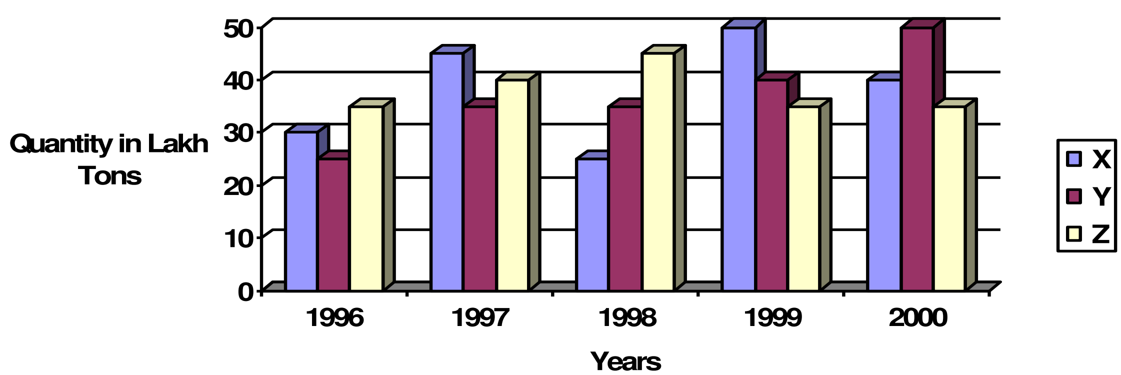
4(e) required ratio=(75+65)/(85+95)=140/180=7/9

5(e)average sales(in thousand numbers of branches B1,B3,and B6 in 2000= 1/3*(80+95+70)=245/3

average sales(in thousand numbers of branches B1,B2,and B3 in 2001=1/3*(105+65+110)=280/3

therefore required percentage=[((245/3)/(280/3))*100]%=(245/280*100)%=87.5%

Ex.3. The bar graph provided below gives the data of the production of paper (in thousand tonnes) by three different companies x,y and z over the years .study the graph and answer the questions that follow
production of paper(in laks tonnes) by three companys x,yand z over the years



1.What is the difference between the production of the company Z in 1998 and company y in 1996?
a.2 ,00,000 tons
b.20,00,000 tons
c.20,000 tons
d.2,00,00,000 tons
e.none of these

2.what is the ratio of the average production of company x in the period 1998 to 2000 to the average production of company y in the same period?
a.1:1
b.15:27
c.23:25
d.27:29

e.none of these

3.what is the percentage increase in the production of company y from1996 to 1999?
a.30%
b.45%
c.50%
d.60%
e.75%

4.the avreage production of five years was maximum for which company?
a.x
b.y
c.z
d.x & y both
e.x and z both

5.for which of thw followlwing years the percentage rise / fall in production from previous year is the maximum for company y?
a.1997
b.1998
c.1999
d.2000
e.1997 & 2000

6.in which year was the percentage of production of company z to the production of company y the maximum?
a.1996
b.1997
c.1998
d.1999
e.2000
Sol: **1**(b):required difference =
[(45-25)*i,00,000]tons=20,00,000 tons

2(c):average production of company x in the period 1998-2000=[1/3*(25+50+40)]=(115/3) lakh tons

average production of company y in the period 1998-2000
[1/3*(35+40+50)]=(125/3) lakh tons

therefore req ratio=(115/3)/(125/3)=115/125=23/25

3(d):percentage increase in the production y from 1996-1999=[(40-25)/25*100]%=(15/25*100)%=60%

4(e):average production (in lakh tons)in five years for the three company's are:
for company x=[1/5*(30+45+25+50+40)]=190/5=38
for company y=[1/5*(25+35+35+40+50)]=185/5=37
for company z=[1/5*(35+40+45+35+35)]=190/5=38
therefore the average production of maximum for both the company's x and z

5(a) : Percentage change (rise/fall)in the production of Company Y in comparison to the previous year, for different years are:

For 1997 = [((32-25)/25)*100]% = 40%

For 1998 = [((35-35)/25)*100]% = 0%

For 1999 = [((40-35)/35)*100]% = 14.29%

For 2000 = [((50-40)/40)*100]% = 25%

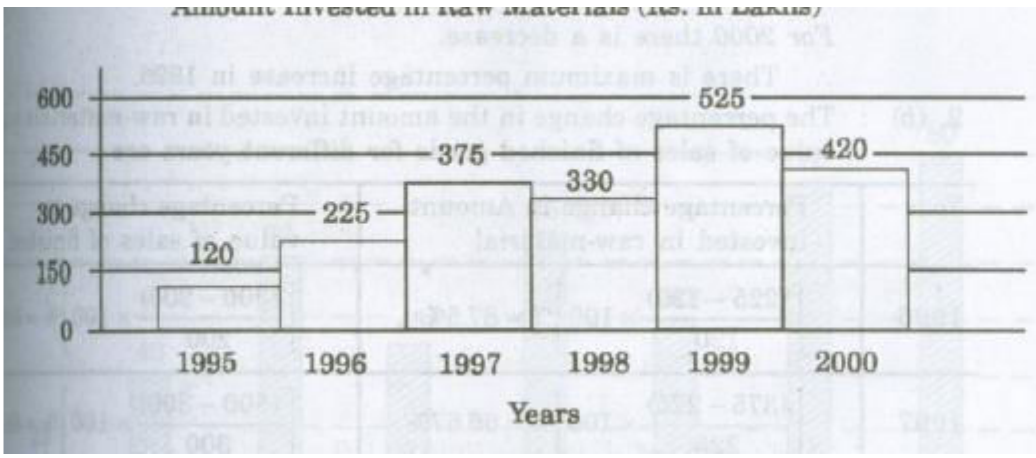
Hence, the maximum percentage rise/fall in the production of company Y is for 1997.

6(a) : The percentages of production of company z to the production of company z for various years are:
For 1996 = ((35/25)*100)%=140%; For 1997 = ((40/35)*100)% = 114.29%
For 1998 = ((45/35)*100)%=128.57%; For 1999 = ((35/40)*100)%=87.5%
For 2000 = ((35/50)*100)%=70%

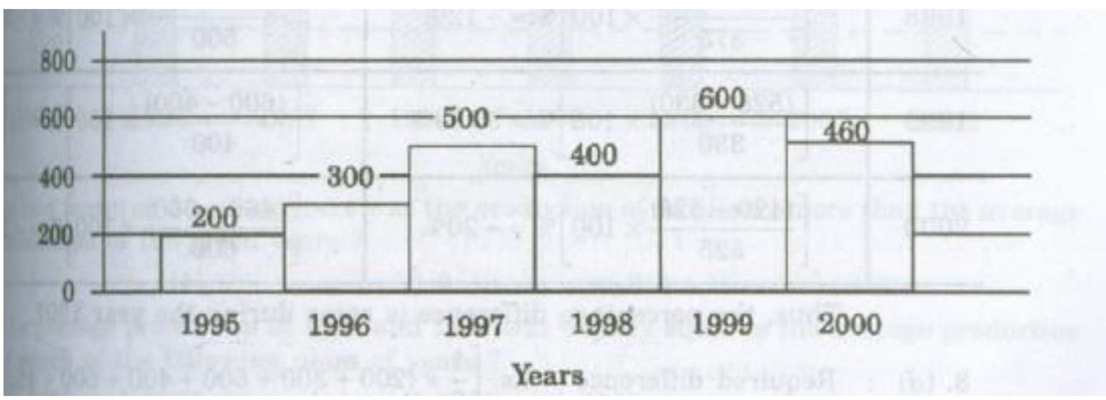
Clearly, this percentage is highest for 1996.

Ex.4.Outof the two bar graphs provided below, one shows the amounts (in Lakh Rs) invested by a Company in purchasing raw materials over the years and the other shows the values(in Lakh Rs.) of finished goods sold by the Company over the years. Study the two bar graphs and answer the questions based on them.
Amount Invested in Raw Materials and the Value of Sales of Finished Goods for a Company over the Years

Amount Invested in Raw Materials (Rs. in Lakhs)



Value of Sales of Finished Goods(Rs.in Lakhs)



1.Inwhichyear,there has been a maximum percentage increase in the amount invested inRawMaterials as compared to the previous year?
*(a)*1996 (b) 1997 *(c)* 1998 (d) 1999 *(e)* 2000
2. Inwhichyear, the percentage change (compared to the previous year) in the investment onRaw Materials is the same as that in the value of sales of finished goods?
*(a)*1996 *(b)* 1997 *(c)* 1998 (d) 1999 *(e)* 2000
3.Whatwas the difference between the average amount invested in Raw Materials during the given period and the average value of sales of finished goods during this period?
*(a)*Rs. 62.5 lakhs *(b)* Rs. 68.5 lakhs *(c)* Rs. 71.5 lakhs
(d)Rs.77.51akhs *(e)* Rs. 83.5 lakhs

4. The value of sales of finished goods in 1999 was approximately what percent of the average amount invested in Raw Materials in the years 1997,1998 and 1999?
(a) 33% (b) 37% (c) 45% (d) 49% (e) 53%

5. The maximum difference between the amount invested in Raw Materials and the value of sales of finished goods was during the year:
(a) 1995 (b) 1996 (c) 1997 (d) 1998 (e) 1999

Sol. 1. (a) : The percentage increase in the amount invested in raw-materials as compared to the previous year, for different years are:
For 1996 = [((225-120)/120)*100]% = 87.5%
For 1997 = [((375-225)/225)*100]% = 66.67%
For 1998 = [((525-330)/330)*100]% = 59.09%
For 2000 there is a decrease.

2.(b)The percentage change in the amount invested in raw-materials and in the value of sales of finished goods for different years are:

year	Percentage change in amount invested in raw-materials	Percentage change in value of sales of finished goods
1996	[((225-120)/120)*100]% = 87.5%	[((300-200)/200)*100]% = 50%
1997	[((375-225)/225)*100]% = 66.7%	[((500-300)/300)*100]% = 66.67%
1998	[((525-330)/330)*100]% = -12%	[((400-500)/500)*100]% = -20%
1999	[((525-330)/330)*100]% = 59.09%	[((600-400)/400)*100]% = 50%
2000	[((420-525)/525)*100]% = -20%	[((460-600)/600)*100]% = -23.33%

Thus the percentage difference is same during the year 1997.

3. (d) : Required difference = Rs. [(1/6)*(200+300+500+400+600+460)
- (1/6)*(120+225+375+330+525+420)] lakhs
= Rs. [(2460/6)-(1995/6)] lakhs = Rs.(410-332.5)lakhs = 77.5 lakhs.

4. (d) : Required percentage = [(600/(375+300+525))*100]% = 48.78% ≈ 49%

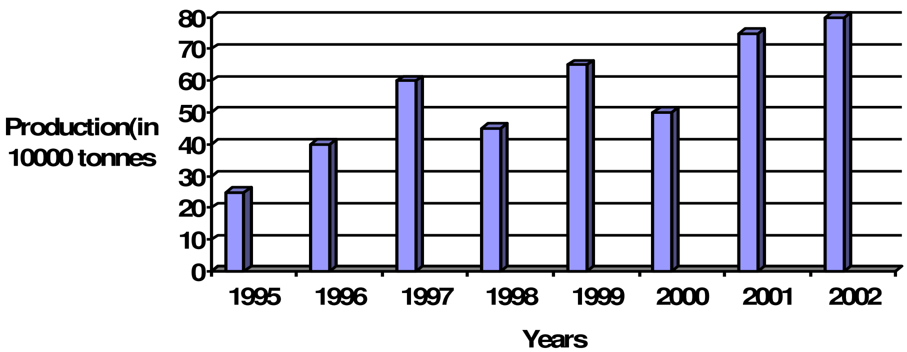
5. (c) : The difference between the amount invested in raw-material and the value of sales of finished goods for various years are :

For 1995 = Rs.(200-120)lakhs = Rs. 80 lakhs
For 1996 = Rs.(200-225)lakhs = Rs. 75 lakhs
For 1997 = Rs. (500-375)lakhs = Rs. 125 lakhs
For 1998 = Rs. (400-330)lakhs = Rs. 70 lakhs.
For 1999 = Rs. (600-525)lakhs = Rs. 75 lakhs
For 2000 = Rs. (460-420)lakhs = Rs. 40 lakhs.
Clearly, maximum difference was during 1997

EXERCISE 37

Directions(questions 1 to 5) : study the following bar-graph and answer the questions given below.
Production of fertilizers by a Company (in 10000 tonnes) over the Years

X axis=years
Y axis=Production (in 10000 tonnes)



1. **In how many of the given years was the production of fertilizers more than the average production of the given years?**

   (a)1 (b)2 (c)3 (d)4 (e)5
2. **The average production of 1996 and 1997 was exactly equal to the average production of which of the following pairs of years?**

   (a)2000 and 2001 (b)1999 and 2000 (c)1998 and 2000 (d)1995 and 1999 (e)1995 and 2001

3.What was the percentage decline in the production of fertilizers from 1997 to 1998?
(a) $33^1/_3$% (b) 30% (c) 25% (d) 21% (e) 20%

4.In which year the percentage increase in production as compared to the previous year the maximum?
(a) 2002 (b) 2001 (c) 1999 (d) 1997 (e) 1996

38. PIE-CHARTS

IMPORTANT FACTS AND FORMULAE

The **pie-chart** or a **pie-graph** is a method of representing a given numerical data in the form of sectors of a circle.
The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding value of the component of the data.
From geometry, we know that the area of a circle is proportional to the central angle.
So, the central angle of each sector must be proportional to the corresponding value of the component.
Since the sum of all the central angle is 360°, we have

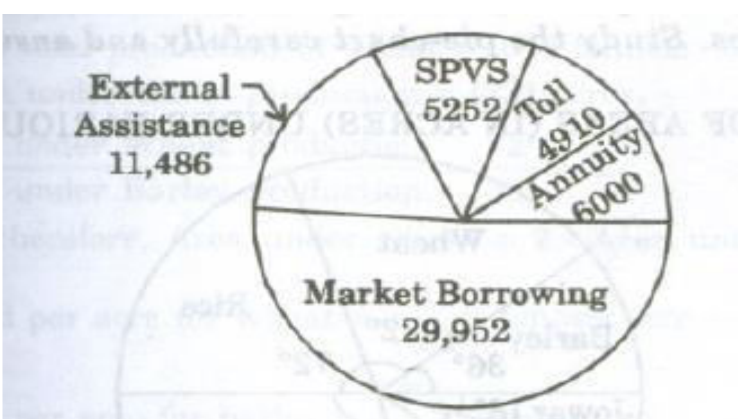
$$\text{Central angle of the component} = \left| \frac{\text{Value of the component}}{\text{Total value}} * 360 \right|^{\circ}$$

SOLVED EXAMPLES

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

Example 1. The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the questions that follow.

SOURCES OF FUNDS TO BE ARRANGED BY NHAI FOR PHASE II PROJECTS (IN CRORES RS.)



Total funds to be arranged for Projects (Phase II) =Rs.57,600 crores.

1. Near about 20% of the funds are to be arranged through:
(a) SPVS (b) External Assistance
(c) Annuity (d) Market Borrowing

2. The central angle corresponding to Market Borrowing is:
(a) 52° (b) 137.8°
(c) 187.2° (d) 192.4°

3. The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is:
(a) 2:9 (b) 1:6
(c) 3:11 (d) 2:5

4. If NHAI could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowings to arrange for the shortage of funds ?
(a) 4.5% (b) 7.5%
(c) 6% (d) 8%

5.If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4910 crores ?
(a) Rs.6213 crores (b) Rs. 5827 crores
(c) Rs. 5401 crores (d) Rs. 5216 crores

SOLUTION

1. (b): 20% of the total funds to be arranged = Rs.(20% of 57600) crores
= Rs.11520 crores Rs.11486 crores.

2. (c):Central angle corresponding to Market Borrowing = $\frac{29952}{57600} * 360°$
= 187.2°

3. (b):
Required ratio = $\frac{4910}{29952} = \frac{1}{6.1} = \frac{1}{6}$

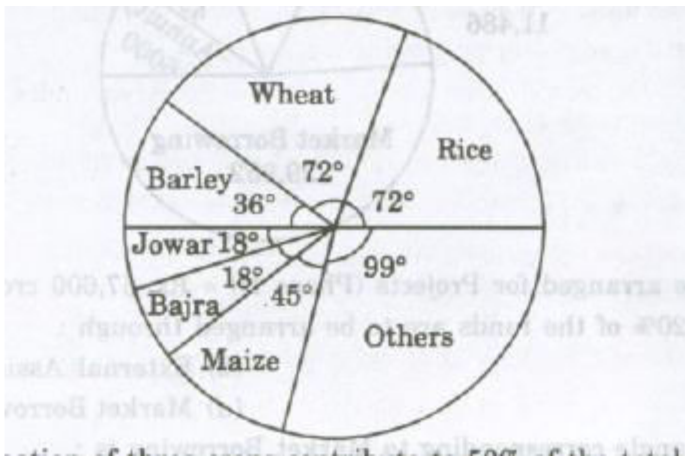
4. (c):Shortage of funds arranged through External Assistance
=Rs.(11486-9695) crores =Rs. 1791 crores.
therefore, Increase required in Market Borrowings =Rs. 1791 crores.

Percentage increase required = $\left| \frac{1791}{29952} * 100 \right|$ % = 5.98 % = 6%

5. (c):Amount permitted = (Funds required from Toll for projects of Phase II) + (10 % of these funds)
=Rs. 4910 crores + Rs. (10% of 4910) crores
=Rs. (4910 + 491) crores = Rs. 5401 crores.

Example 2. The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.

DISTRIBUTION OF AREAS (IN ACRES) UNDER VARIOUS FOOD CROPS



1.Which combination of three crops contribute to 50% of the total area under the food crops ?
(a) Wheat, Barley and Jowar (b)Rice, Wheat and Jowar
(c) Rice, Wheat and Barley (d)Bajra, Maize and Rice

2.If the total area under jowar was 1.5 million acres, then what was the area (in million acres) under rice?
(a)6 (b)7.5 (c)9 (d)4.5

3.If the production of wheat is 6 times that of barley, then what is the ratio between the yield per acre of wheat and barley ?
(a) 3:2 (b) 3:1 (c) 12:1 (d) 2:3

4.If the yield per acre of rice was 50% more than that of barley, then the production of barley is what percent of that of rice ?
(a)30% (b)33 % (c)35% (d)36%

5.If the total area goes up by 5%, and the area under wheat production goes up by 12% , then what will be the angle for wheat in the new pie-chart ?
(a) 62.4° (b) 76.8° (c) 80.6° (d) 84.2°

SOLUTIONS

1.(c):The total of the central angles corresponding to the three crops which cover 50% of the total area ,should be 180°.Now, the total of the central angles for the given combinations are:

(i) Wheat,Barley and jowar = $(72°+36°+18°)=126°$
(ii) Rice,Wheat and Jowar = $(72°+72°+18°)=162°$
(iii) Rice,Wheat and Barley = $(72°+72°+36°)=180°$
(iv)Bajra,Maize and Rice = $(18°+45°+72°) = 135°$
Clearly:(iii) is the required combination.

2.(a): The area under any of the food crops is proportional to the angle corresponding to that crop.
Let the area under the rice production be x million acres.
Then, $18:72 = 1.5:x \Rightarrow x=(72*15/18)=6$
Thus, the area under rice production be = 6 million acres.

3.(b): Let the total production of barley be T tones and let Z acres of land be put under barley production.
Then, the total production of wheat =(6T) tones.
Also,area under wheat production = (2Z) acres.

$$\therefore \frac{\text{Area Under Wheat Production}}{\text{Area Under Barley Production}} = \frac{72°}{36°} = 2$$

And therefore,Area under wheat = 2*Area under Barley = (2Z)acres

Now, yield per acre for wheat = (6T/2Z) tones/acre = (3T/Z) tones/acre
And yield per acre for barley = (T/Z) tones/acre.

$$\therefore \text{Required ratio} = \frac{3T/Z}{T/Z} = 3:1.$$

4. (b):Let Z acres of land be put under barley production.

$$\text{Then, } \frac{\text{Area Under Rice Production}}{\text{Area Under Barley Production}} = \frac{72°}{36°} = 2.$$

$\therefore$ Area under rice production = 2 * area under barley production = (2Z) acres.
Now,if p tones be the yield per acre of barley then ,yield per acre of rice
=(p+50% of p) tones =(3/2 p) tones.
$\therefore$ Total production of rice = (yield per acre) * (area under production)
= (3/2 p)*2Z=(3pZ) tones.
And,Total production of barley = (pz) tones.
$\therefore$ Percentage production of barley to that rice = (pZ/3pZ *100)%= 33 1/3% .

5.(b): Initially,let t be the total area under considerations.
The area under wheat production initially was =(72/360 * t)acres = (t/5)acres.
Now,if the total area under consideration be increased by 5%,
then the new value of the total area= (105/100 t) acres.
Also,if the area under wheat production be increased by 12%,

$$\text{then the new value of area under wheat} = \left| \frac{t}{5} + \left(12\% \text{ of } \frac{t}{5}\right) \right| \text{ acres} = (112t/500)\text{acres.}$$

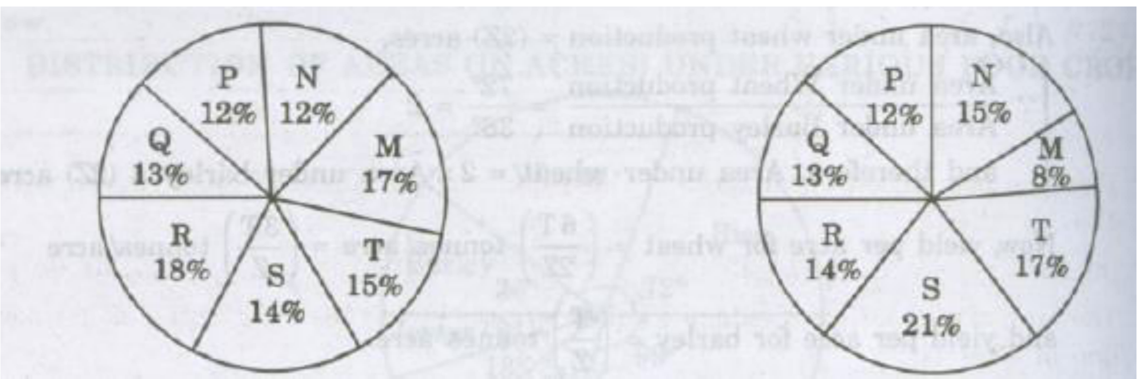
∴ Central angle corresponding to wheat in the pie-chart

$$= \left[\frac{\text{Area Under Wheat (new)}}{\text{Total area (new)}} * \mathbf{360}\right]^\circ = \left|\frac{(112t/500)}{(105t/100)} * 360\right|^\circ = 76.8^\circ$$

Example 3.The following pie-charts show the distribution of students of graduate and post graduate levels in seven different institute-M,N,P,Q,R,S and T in a town.

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES-M,N,P,Q,R,S AND T.

Total Number of students of graduate level | **Total Number of students of post graduate level**



1.How many students of institutes M and S are studying at graduate level?
(a) 7516 (b) 8463 (c) 9127 (d) 9404

2.Total number of students studying at post -graduate level from institutes N and P is:
(a) 5601 (b) 5944 (c) 6669 (d) 7004

3.What is the total number of graduate and post-graduate level students in institute R?
(a) 8320 (b) 7916 (c) 9116 (d) 8372

4 .What is the ratio between the number of students studying at post graduate and graduate levels respectively from institute S?
(a) 14:19 (b) 19:21 (c) 17:21 (d) 19:14

5.What is the ratio between the number of students studying post graduate level from institute S and the number of students studying at graduate level from institute Q?
(a) 13:19 (b) 21:13 (c) 13:8 (d)19:13

SOLUTION

1.(b):Students of institute M at graduate level = 17% of 27300 = 4641.
Students of institute S at graduate level = 14% of 27300 = 3822
$\therefore$ Total number students at graduate level in institutes M and S = 4641+3822=8463

2.(c):Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.

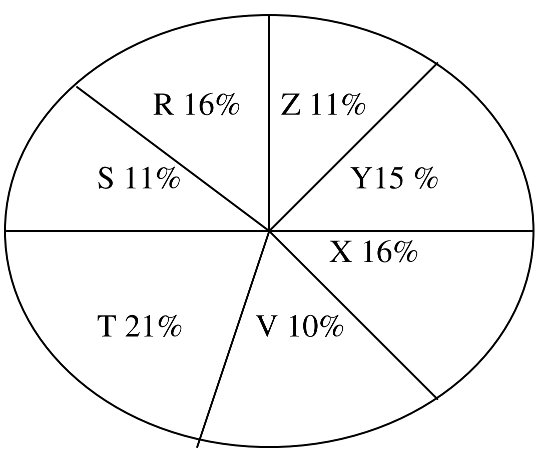
3.(d):Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.

4.(d):Required ratio = $\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{21 * 24700}{14 * 27300} = \frac{19}{14}$

5.(d):Required ratio = $\frac{(21\% \text{ of } 24700)}{(13\% \text{ of } 27300)} = \frac{21 * 24700}{13 * 27300} = \frac{19}{13}$

Example 4.Study the following pie-chart and the table and the answer the questions based on them.

PROPORTION OF POPULATION OF SEVEN VILLAGES IN 1997



village	% population below poverty
X	38
Y	52
Z	42
R	51
S	49
T	46
V	58

1.Find the population of villages S if the population of village X below poverty line in 1997 is 12160.
(a). 18500 (b) 20500 (c) 22000 (d) 26000

2.The ratio of the population of the village T below poverty line to that of village Z below poverty line in 1997 is:
(a) 11:23 (b) 13:11 (c) 23:11 (d) 11:13

3.If the population of village R in 1997 is 32000,then what will be the population of village Y below poverty line in that year?

(a) 14100 (b)15600 (c) 16500 (d) 17000

4.If in 1998, the population of villages Y and V increases by 10% each and the percentage of population below poverty line remains unchanged for all the villages, then find the population of village V below poverty line in 1998,given that the population of village Y in 1997 was 30000.

(a) 11250 (b) 12760 (c) 13140 (d) 13780

5.If in 1998,the population of village R increases by 10% while that of village Z reduces by 5% compared that in 1997 and the percentage of population below poverty line remains unchanged for all the village,then find the approximate ratio of population of village R below poverty line for the year 1999.

(a) 2:1 (b) 3:2 (c) 4:3 (d) 5:4

SOLUTION

1.(c):Let the population of village X be x

Then,38% of x=12160 $\Rightarrow$ $x = \frac{12160 * 100}{38}$ =3200

Now ,if s be the population village S,then

16:11 = 32000 : s $\Rightarrow$ $s = \frac{11 * 32000}{16}$ = 22000.

2.(c): Let N be the total population of all the seven villages.

Then ,population of village T below poverty line = 46% of (21% of N) and population of village Z below poverty line = 42% of (11% of N)

$$\therefore \text{Required ratio} = \frac{46\% \text{ of } (21\% \text{ of N})}{42\% \text{ of } (11\% \text{ of N})} = \frac{46 * 21}{42 * 11} = \frac{23}{11}$$

3.(b): Population of village R = 32000(given)

Let the population of village Y be y.

Then, 16:15 = 32000 : y $\Rightarrow$ $y = \frac{15 * 32000}{16}$ =30000

4.(b): Population of village Y in 1997 = 30000(given) .

Let the population village V in 1997 be v.

Then, 15:10 = 30000:v $\Rightarrow$ $v = \frac{30000 * 10}{15}$ = 20000.

Now population of village V in 1998 = 20000 + (10% 0f 20000) = 20000.

$\therefore$Population of village V below poverty line in 1998 = 58% of 22000 = 12760.

5.(a) : Let the total population of all the seven villages in 1997 be N.

Then,population of village R in 1997 = 16% of N = 16/ 100 N

And population of village Z in 1997 = 11% of N =11/100 N

$\therefore$Population of village R in 1999 = {16/100N+(10% of 16/100 N)}=1760/10000 N

and population of village Z in 1999 = {11/100 N-(5% of 11/100 N)} = 1045/10000 N.
Now,population of village R below poverty line for 1999 = 51% of(1760/10000 N)
And population of village Z below poverty line 1999 = 42% of (1045/10000 n)

$$\text{Required ratio} = \frac{51\%\ \text{of}\ (1760/10000\ N)}{42\%\ \text{of}\ (1045/10000\ N)} = \frac{51 * 1760}{42 * 1045} = \frac{2}{1}.$$

39.LINE-GRAPHS

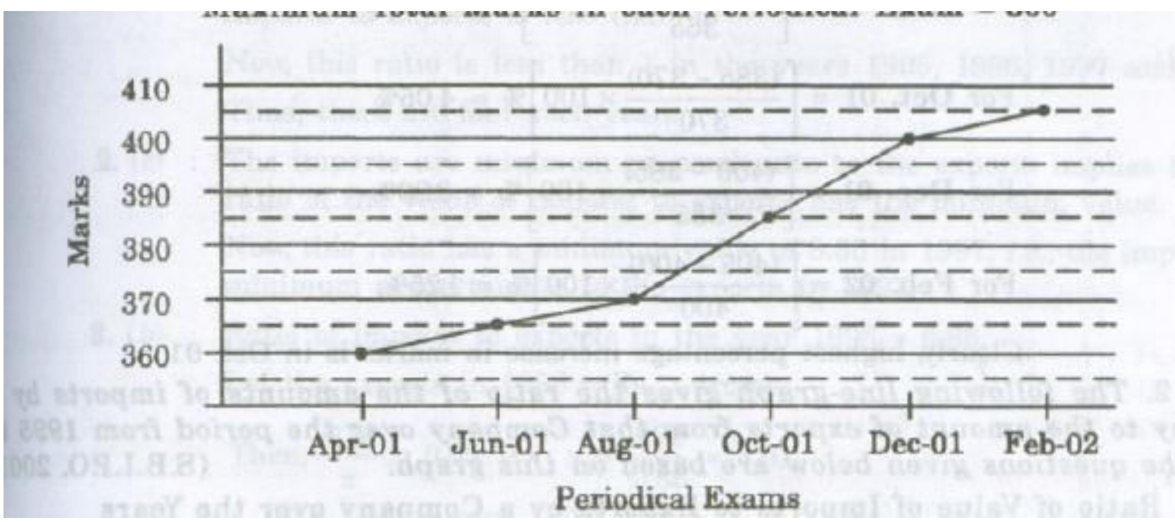
This section comprises of question in which the data collected in a particular discipline are represented by specific points together by straight lines. The points are plotted on a two-dimensional plane taking one parameter on the horizontal axis and the other on the vertical axis. The candidate is required to analyse the given information and thereafter answer the given questions on the basis of the analysis of data.

SOLVED EXAMPLES

Ex. 1. In a school the periodical examination are held every second month. In a session during Apr. 2001 – Mar. 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below. Study the graph and answer the questions based on it.
(S.B.I.P.O 2003)

MARKS OBTAINED BY A STUDENT IN SIX PERIODICAL EXAMS HELD IN EVERY TWO MONTHS DURING THE YEAR IN THE SESSION 2001-02

Maximum Total Marks In each Periodical Exam = 500



1. **The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in Apr. 01?**
   (a) 110% (b) 112.5% (c) 115% (d) 116.5% (e) 117.5%

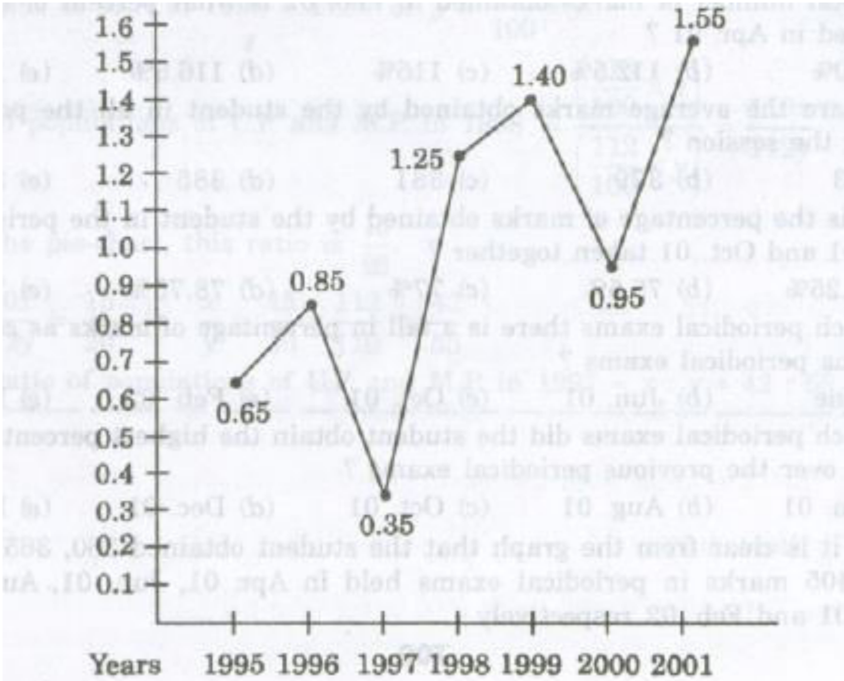
2. **What are the average marks obtained by the student in all the periodical exams of during the session.**
   (a) 373 (b) 379 (c) 381 (d) 385 (e) 389
3. **what is the percentage of marks obtained by the student in the periodical exams of Aug. 01 and Oct. 01 taken together?**
   (a) 73.25% (b) 75.5% (c) 77% (d) 78.75% (e) 79.5%
4. **In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams?**
   (a) None (b) Jun. 01 (c) Oct. 01 (d) Feb. 01 (e) None of these
5. **In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams?**
   (a) Jun. 01 (b) Aug. 01 (c) Oct. 01 (d) Dec. 01 (e) Feb. 02

Sol. Here it is clear from the graph that the student obtained 360, 365, 370, 385, 400 and 405 marks in periodical exams held in Apr. 01, Jun. 01, Aug. 01, Oct. 01, Dec. 01 and Feb. 02 respectively.

1. (b) : Required percentage = [(405/360)*100] % = 112.5 %
2. (c) : Average marks obtained in all the periodical exams.
   = (1/6)*[360+370+385+400+404] = 380.83 ≈ 381.
3. (d) : Required percentage = [(370+385)/(500+500) * 100] % = [(755/1000)*100]% =75.5%
4. (a) : As is clear from graph, the total marks obtained in periodical exams, go on increasing. Since, the maximum marks for all the periodical exams are same , it implies that the percentage of marks also goes on increasing. Thus, in none of the periodical exams, there is a fall in percentage of marks compared to the previous exam.
5. (c) : Percentage increases in marks in various periodical exams compared to the previous exams are:
   For **Jun. 01** = [(365-360)/360 * 100] % = 1.39 %
   For **Aug. 01** = [(370-365)/365 * 100] % = 1.37 %
   For **Oct. 01** = [(385-370)/370 * 100] % = 4.05%
   For **Dec. 01** = [(400-385)/385 * 100] % = 3.90 %
   For **Feb. 02** = [(405-400)/400 * 100] % = 1.25 %

Ex. 2. The following line- graph the ratio of the amounts of imports by a Company to the amount of exports from that Company over the period from 1995 to 2001. The questions given below are based on this graph. (S.B.I.P.O 2001)

Ratio of value of Import to Export by a Company over the Years



1. **In how many of the given years were the exports more than the imports?**
   a. 1 b. 2 c. 3 d. 4

2.The imports were minimum proportionate to the exports of the Company in the year:
a..1995 b.1996 c.1997 d.2001

3.If the imports of the Company in 1996 was Rs.272 crores, the exports from the Company in 1996 was:
a. Rs.370 crores b.Rs.320 crores c.Rs.280 crores
d.Rs.275 crores e.Rs.264 crores

4.What was the percentage increase in imports from 1997 to 1998?
a. 72 b.56 c.28 d.None of these e.Data inadequate

5.If the imports in 1998 was Rs.250 crores and the total exports in the years 1998 and 1999 together was Rs.500 crores, then the imports in 1999 was:
a.Rs.250 crores b.Rs.300 crores c.Rs 357 crores
d.Rs 420 crores e.None of these

Sol: 1. d : The exports are more than the imports implies that the ratio of value of imports to exports is less than 1.
Now, this ratio is less than 1 in the years 1995,1996,1997 and 2000.
Thus, there are four such years.

2. c: The imports are minimum proportionate to the exports implies that the ratio of the value of imports to exports has the minimum value.

Now, this ratio has a minimum value of 0.35 in 1997, i.e., the imports are minimum proportionate to the exports in 1997.

3. b: Ratio of imports to exports in the years 1996=0.85.

Let the exports in 1996=Rs.320 crores.

Then,272/x =0.85 implies x=272/.85 = 320.

Exports in 1996 = Rs.320 crores.

4. e: The graph gives only the ratio of imports to exports for different years. To find the percentage increase in imports from 1997 to 1998, we require more details such as the value of imports or exports during these years. Hence, the data is inadequate to answer this question.

5. d: The ratio of imports to exports for the years 1998 and 1999 are 1.25 and 1.40 respectively.

Let the exports in the year 1998 = Rs. x crores

Then, the exports in the year 1999=Rs(500-x) crores.

1.25=250/x implies x=250/1.25=200

Thus the exports in the year 1999=Rs. (500-200)crores=Rs.300 crores

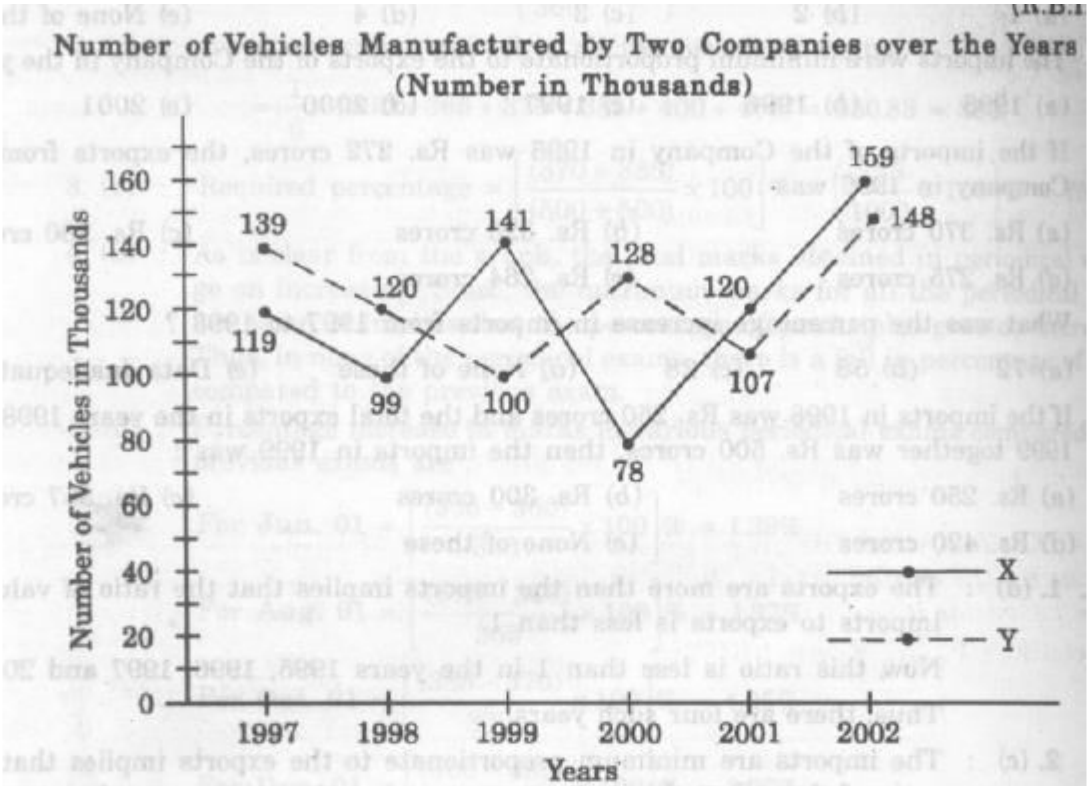
Let the imports in the year 1999=Rs. y crores

Then, 1.4=y/300 implies y=(300*1.4)=420.

Imports in the year 1999=Rs.420 crores.

Ex.3.Study the following line-graph and answer the question based on it.

Number of vehicle Manufactured by Two Companies over the Years
(Numbers in thousands)



1.What is the difference between the total productions of the two Companies in the given years?

a. 19000 b. 22000 c.26000 d.28000 e.29000

2.What is the difference between the numbers of vehicles manufactured by Company Y in 2000 and 2001?

a.50000 b.42000 c.33000 d.21000 e.13000

3.What is the average number of vehicles manufactured by Company X over the given period? (rounded off to the nearest integer)

a.119333 b.113666 c.112778 d.111223 e. None of these

4.In which of the following years, the difference between the productions of Companies X and Y was the maximum among the given years?

a.1997 b.1998 c.1999 d.2000 e.2001

5.The production of Company Y in 2000 was approximately what percent of the production of Company X in the same year?

a.173. b.164 c.132 d.97 e.61

Sol: From the line-graph it is clear that the productions of Company X in the years 1997,1998,1999,2000,2001 and 2002 are 119000,99000,141000,78000,120000and 159000 respectively and those of Company Y are 139000,120000,100000,128000,107000 and 148000 respectively.

1. (c) : Total production of Company X from 1997 to 2002
   = 119000+99000+141000+78000+120000+159000 = 716000
   and total production of Company Y from 1997 to 2002
   =139000+120000+100000+128000+107000+148000=742000
   Difference=742000-716000=26000.

2. (d) : Require difference = 128000-107000 = 21000.
3. (a) : Average number of vehicles manufactured by Company X
   = (91/6)* (119000 + 99000 + 141000 + 78000 + 120000 + 159000) = 119333.

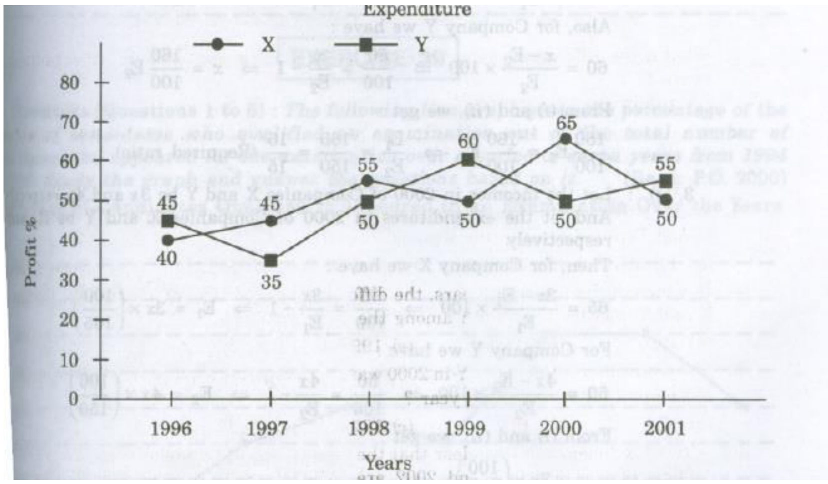
4. (d) : The difference between the production of Companies X and Y in various years are.
   For 1997 = (139000 – 119000) = 20000;
   For 1998 = (120000 – 99000) = 21000;
   For 1999 = (141000 – 100000) = 41000;
   For 2000 = (128000 – 78000) = 50000;
   For 2001 = (120000 – 107000) = 13000;
   For 2003 = (159000 – 148000) = 11000;
   Clearly, maximum difference was in 2000.

5. (b) : Required percentage = [(128000/78000)* 100] % = 164 %.

Ex. 4. The following line-graph gives the percent profit earned by two Companies X and Y during the period 1996 – 2001. Study the line – graph and answer the questions that are based on on it.

Percentage Profit Earned by Two Companies X and Y over the Given years

% profit/ loss = [(Income – Expenditure) / Expenditure] * 100



1. **If the expenditure of Company Y in 1997 was Rs. 220 crores, what was its income in 1997?**

(a). Rs. 312 crores (b). Rs. 297 crores (c) Rs. 283 crores (d) Rs. 275 crores (e)Rs.261 crores

2.If the incomes of the two companies were equal in 1999,then what was the ratio of expenditure of Company X to that of company Y in 1999 ?

(a) 6:5 (b) 5:6 (c) 11:6 (d) 16:15 (e) 15:16

3.The incomes of the companies X and Y in 2000 were in the ratio of 3:4 respectively.What was the respective ratio of their expenditures in 2000?

(a) 7:22 (b) 14:19 (c) 15:22 (d)27:35 (e) 33:40

4.If the expenditure ofcompanies X and Y in 1996 were equal and the total income of the two companies in 1996 was Rs.342 crores, what was the total profit of the twocompanies together in 1996 ? (Profit = Income – Expenditure)

(a) Rs.240crores (b) Rs.171crores (c) Rs.120crores (d) Rs.102crores (e)None of these.

5.The expenditure ofcompany X in the year 1998 was Rs.200 crores and the income Company X in 1998 was the same as its expenditure in 2001 was:

(a) Rs.465crores (b)Rs.385crores (c)Rs.295crores (d)Rs.255crores

Sol.1.(b) : Profit percent ofcompany Y in 1997=35.

Let the income of company Y in 1997 be Rs.x crores

Then,$35 = \frac{x-220}{220} \times 100 \Rightarrow x = 297$

$\therefore$ Income of company Yin 1997 = Rs.297crores

2.(d): Let the incomes of the twocompanies X and Yin 1999 be Rs.x and let the Expenditures of companies X and Y in 1999 be E_1 and E_2 respectively

Then, for Company X we have:

$$50 = \frac{x-E1}{E1} \times 100 \Rightarrow \frac{50}{100} = \frac{x}{E1} - 1 \Rightarrow x = \frac{150}{100} E1$$

Also, for the Company Y we have:

$$60 = \frac{x-E2}{E2} * 100 \Rightarrow \frac{60}{100} = \frac{x}{E2} - 1 \Rightarrow x = \frac{160}{100} E2$$

From (i) and (ii),we get

$$\frac{150}{100} E1 = \frac{160}{100} E2 \Rightarrow \frac{E1}{E2} = \frac{160}{150} = \frac{16}{15} \text{(Required ratio)}$$

3.(c):Let the incomes in 2000 of companies X and Y be 3x and 4x respectively.And let the expenditure in 2000 of companies X and Y be E1 and E2 respectively.

Then, for company X we have:

$$65 = \frac{3x-E1}{E1} * 100 \Rightarrow \frac{65}{100} = \frac{3x}{E1} - 1 \Rightarrow E1 = 3x * \left(\frac{100}{165}\right)$$

For company Y we have:

$$50 = \frac{4x-E2}{E2} * 100 \Rightarrow \frac{50}{100} = \frac{4x}{E2} - 1 \Rightarrow E2 = 4x * \left(\frac{100}{150}\right)$$

From (i)and(ii) we get:

$$\frac{E1}{E2} = \frac{3x*(100/165)}{4x*(100/150)} = \frac{3*150}{4*165} = \frac{15}{22} \text{(Required ratio)}$$

4.(d):Let the expenditures of each of the Companies X and Y in 1996 be Rs.xcrores.And let the income of Company X in 1996 be Rs.zcrores so that the income of Company Y in 1996 =Rs.(342-z)crores.

Then,for company X we have:

$$40 = \frac{z-x}{x} * 100 \Rightarrow \frac{40}{100} = \frac{z}{x} - 1 \Rightarrow x = \frac{100z}{140}$$

Also for company Y we have:

$$45 = \frac{(342-z)-x}{x} * 100 \Rightarrow \frac{45}{100} = \frac{(342-z)}{x} - 1 \Rightarrow x = \frac{(342-z)*100}{145}$$

From(i)and (ii) we get:

$$\frac{100z}{140} = \frac{(342-z)*100}{145} \Rightarrow z = 168$$

Substituting z=168 in (i),we get: x=120

∴ Total expenditure of companies X and Y in 1996=2x=Rs.240crores.

Total income of companies X and Y in 1996=Rs.342 crores.

∴Total profit =Rs.(342-240)crores =Rs.102 crores

5.(a): Let the income of company X in 1998 be Rs.x crores.

Then, $55 = \frac{x-200}{200} * 100 \Rightarrow x = 310.$

$\therefore$ Expenditure of Company X in 2001= Income of company X in 1998 = Rs.310crores

Let the income of company X in 2001 be Rs.z crores

Then, $50 = \frac{z-310}{310} * 100 \Rightarrow z = 465.$

$\therefore$ Income of company X in 2001 = Rs.465 crores.