Booklet No. A/B 0398

2019

Mathematical Science

PAPER – II

(Group A and B)

Full marks – 200

Time – 2 hours [11.30 a.m. to 1.30 p.m.]

Signature and Name of Invigilator

(Signature) B Talukdar

(Name) Barnali Talukdar.

Time : **2** hours] [Maximum Marks : **200**

Number of Pages in this Booklet : **52** Number of Questions in this Booklet : **160**

Instructions for the Candidates

1. Please **write your subject of PG (Mathematics / Statistics) in the space marked as (subject for examination) in the Response (OMR) Sheet.**
2. This paper consists of **one hundred and sixty (160)** objective type questions. Out of which **forty (40)** questions are **common to both the candidates of Mathematics and Statistics**. Out of the rest **one hundred twenty (120)** questions, the **Mathematics candidates** will attempt **Group–A** consisting of **sixty (60)** questions and **Statistics candidates** will attempt **Group–B** consisting of **sixty (60)** questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are required to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the Booklet with the information printed on the cover page. Faulty Booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : Ⓐ Ⓑ ● Ⓓ
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the **OMR Sheet given with the Booklet only**. If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated. **Also, answer once marked shall not be cancelled or erased. If anybody does it, his/her OMR Sheet shall be treated as cancelled and shall not be evaluated.**
6. Read instructions given above carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or resort to any other unfair means, you will render yourself liable to disqualification.
9. You have to return the Original OMR Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Question Booklet and the carbon copy of OMR Sheet on conclusion of examination.
10. **Use only Ball point pen.**
11. **Use of any calculator or log table etc., is prohibited.**
12. **There is no negative marks for incorrect answers.**

SEAL

MATHEMATICAL SCIENCE

PAPER – II

Note : This paper consists of **one hundred and sixty (160)** objective type questions. Out of which **forty (40)** questions are **common to both the candidates of Mathematics and Statistics** each question carrying **two (02)** marks. Out of the rest **one hundred twenty (120)** questions, the **Mathematics candidates** will attempt **Group–A** consisting of **sixty (60)** questions each question carrying **two (02)** marks and **Statistics candidates** will attempt **Group–B** consisting of **sixty (60)** questions each question carrying **two (02)** marks. Answer from **either of the Groups.**

1. Identify from below the one which is NOT TRUE :

   (A) A function which is continuous throughout a closed interval is bounded therein.

   (B) A continuous function in an interval attains its upper and lower bounds, at least once in the interval.

   (C) If a function is continuous at any point then a finite derivative of the function exists at that point.

   (D) Product of two continuous function is a continuous function.

2. Match Group A and Group B and select the correct match from the codes given below :

Group A	Group B
(a) The function $f : R \to R$ defined $f(x) = x^3 - x$ is	(i) a uniformly continuous
(b) The function $f : R \to R$ defined by $f(x) = e^x$ is	(ii) discontinuous at $x = 0$
(c) The function $f : [1, \infty) \to R$ given by $f(x) = \sqrt{x}$ is	(iii) onto, but not 1–1
(d) The function $f(x) = x \sin\frac{1}{x}$, $x \neq 0$ and $f(0) = a$, $a \neq 0$ is	(iv) 1–1, but not onto

Codes :	(a)	(b)	(c)	(d)
(A)	(iii)	(iv)	(ii)	(i)
(B)	(iv)	(iii)	(i)	(ii)
(C)	(iii)	(iv)	(i)	(ii)
(D)	(iv)	(iii)	(ii)	(i)

3. Let γ be any positively oriented circle with centre at origin. Then the value of the integral $\int_{\gamma} \frac{\cos z}{z^2} dz$ is

(A) $2\pi i$

(B) $-2\pi i$

(C) 0

(D) πi

4. The eigenvalues of

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

are

(A) 2, 2, 4

(B) −2, −2, 4

(C) 2, 2, −4

(D) −2, −2, −4

5. Which of the following sets is not a subspace of $M_n(F)$, the vector space of $n \times n$ matrices over a field F?

(A) All upper triangular matrices of order n

(B) All non-singular matrices of order n

(C) All symmetric matrices of order n

(D) All matrices of order n, the sum of whose diagonal entries is zero.

6. The matrix A has eigenvalues 1 and 4 with corresponding eigenvectors $(1, -1)^T$ and $(2, 1)^T$ respectively. The matrix A is

(A) $\begin{pmatrix} 3 & 2 \\ 1 & 3 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$

7. Consider the following statements:

(i) For any two 2×2 symmetric matrices A, B; $A^T B = AB^T$

(ii) For a 2×2 non-singular matrix A, $Adj(Adj\,A) = A$

(iii) The inverse of the matrix $\begin{pmatrix} -2 & 3 \\ 5 & -4 \end{pmatrix}$ is $\begin{pmatrix} -4 & 3 \\ 5 & -2 \end{pmatrix}$

(iv) If A is a nilpotent matrix of order n, then $I_n - A$ is a singular matrix

Which of the following is true?

(A) (i) and (ii) are correct

(B) (i) and (iv) are correct

(C) (i), (ii) and (iv) are correct

(D) (ii) and (iii) are correct.

8. The mathematical expectation of the sum of points on n dice is

(A) $\frac{7}{4}n$

(B) $\frac{7}{2}n$

(C) $\frac{4}{7}n$

(D) $\frac{2}{7}n$

9. A system of m linear equations in n unknowns has at most

(A) m basic feasible solutions

(B) n basic feasible solutions

(C) $m+n$ basic feasible solutions

(D) ${}^{n}C_{m}$ basic feasible solutions.

10. The following are given statements :

(a) The real line R is a complete metric space

(b) $C[a, b]$ is an incomplete metric space

(c) $X(0,1)$, where $\rho(x_1, x_2) = |x_1 - x_2|; x_1, x_2 \in X$ is a complete metric space

(d) The set of rational numbers X where $\rho(x_1, x_2) = |x_1 - x_2|; x_1, x_2 \in X$ is an incomplete metric space.

Use the truth table to find the correct answer.

Codes :	(a)	(b)	(c)	(d)
(A)	True	True	False	False
(B)	False	True	True	False
(C)	True	False	False	True
(D)	False	False	True	True

10. Let the bivariate distribution function of the random variables X, Y be denoted by $F(x, y)$. Then the probability content in the region $x_1 \le X \le x_2$, $y_1 \le Y \le y_2$ is given by

$$P(x_1 \le X \le x_2, y_1 \le Y \le y_2) = \int_{x_1}^{x_2}\int_{y_1}^{y_2} f(x, y)\,dx\,dy$$

which is efficiently computed as

(A) $P(x_1 \le X \le x_2,\ y_1 \le Y \le y_2) =$

$$F(x_1, y_1) - F(x_1, y_2) + F(x_2, y_1) - F(x_2, y_2)$$

(B) $P(x_1 \le X \le x_2,\ y_1 \le Y \le y_2) =$

$$F(x_1, y_1) + F(x_2, y_1) + F(x_1, y_2) - F(x_2, y_2)$$

(C) $P(x_1 \le X \le x_2,\ y_1 \le Y \le y_2) =$

$$F(x_1, y_1) - F(x_1, y_2) - F(x_2, y_1) + F(x_2, y_2)$$

(D) $P(x_1 \le X \le x_2,\ y_1 \le Y \le y_2) =$

$$F(x_1, y_1) + F(x_1, y_2) + F(x_2, y_1) + F(x_2, y_2)$$

where $f(x, y)$ is the bivariate probability density function. Identify the correct answer.

11. Let (X,d) be an infinite metric space such that for all $x, y \in X$, $d(x, y) \in \mathbb{N}$. Consider the following statements :

(i) Every open set in (X, d) is infinite

(ii) Every sequence in (X, d) is convergent

(iii) Every singleton in (X, d) is open

(iv) Every singleton in (X, d) is closed

Which of the following is true ?

(A) (i) and (ii) are correct

(B) (iii) and (iv) are correct

(C) (i) and (iii) are correct

(D) (ii) and (iii) are correct

ALT

11. Let A and B be two events belonging to a sample space Ω. Then the probability that exactly one of the events A and B occurs is

(Choose the correct one from below) :

(A) $P(A)+P(B)-P(A\cap B)$

(B) $P(A)+P(B)-2P(A\cap B)$

(C) $min(P(A), P(B))$

(D) $\frac{1}{2}(min(P(A), P(B))+\frac{1}{2}(max(P(A), P(B))$

12. Match the functions of List I with those of List II and choose the correct match from the codes given below :

List I	List II
1. $\frac{\sin\sqrt{z}}{\sqrt{z}}$	(i) essential singularity at $z=0$
2. $e^{1/z}$	(ii) removable singularity
3. $\cot z$	(iii) pole of order 4
4. $\frac{\log_e(z-2)}{(z^2+2z+2)^4}$	(iv) simple pole

Codes :

	1	2	3	4
(A)	(i)	(ii)	(iii)	(iv)
(B)	(ii)	(i)	(iv)	(iii)
(C)	(ii)	(i)	(iii)	(iv)
(D)	(iv)	(ii)	(iii)	(i)

ALT

12. What is the probability that among 'n' people there are at least two people who have the same birthday ?

(A) $\frac{364}{365}$

(B) $1-\frac{(365)_n}{(365)^n}$

(C) $\frac{(364)^n}{(365)^n}$

(D) $\frac{(364)^n}{(365)_n}$

13. Let $f : \mathbb{C} \to \mathbb{C}$ be an analytic function. For $z = x + iy$, let $u, v : \mathbb{R}^2 \to \mathbb{R}$ be such that $u(x, y) = R_e f(z)$ and $v(x, y) = I_m f(z)$.
Which of the following is correct?

(A) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} = 0$

(B) $\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} = 0$

(C) $\dfrac{\partial^2 u}{\partial x \partial y} - \dfrac{\partial^2 u}{\partial y \partial x} = 0$

(D) $\dfrac{\partial^2 v}{\partial x \partial y} + \dfrac{\partial^2 v}{\partial y \partial x} = 0$

ALT

13. Considering Assertion (Ass) and Reasoning (R), choose the correct answer from the given codes :

Assertion (Ass) : In a normal distribution $\beta_1 = 0$

Reason (R) : Because in a normal distribution all odd ordered central moments vanish

Codes :

(A) (Ass) is false but (R) is true

(B) (Ass) is true but (R) is false

(C) Both (Ass) and (R) are false

(D) Both (Ass) and (R) are true

14. Fixed points of the transformation $w = \dfrac{z-1}{z+1}$ are

(A) $1, -1$

(B) $-i, i$

(C) $1, -i$

(D) $-1, i$

ALT

14. Table A provides the density function of a continuous random variable X and Table B contains the expected values. Match the codes of Table A and Table B and select the appropriate options.

Table A		Table B	
1.	$\dfrac{1}{B(\alpha,\beta)} \dfrac{x^{\alpha-1}}{(1+x)^{\alpha+\beta}}$	(a)	$\alpha\beta$
2.	$\dfrac{\beta^\alpha}{\lceil\alpha} e^{-\beta x} x^{\alpha-1}$	(b)	$\dfrac{\alpha}{\beta-1}$
3.	$\dfrac{1}{\lceil\alpha\, \beta^\alpha} e^{\frac{x}{\beta}} x^{\alpha-1}$	(c)	$\dfrac{\alpha}{\beta}$
4.	$\dfrac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$	(d)	$\dfrac{\alpha}{(\alpha+\beta)}$

Codes :	1	2	3	4
(A)	(a)	(b)	(d)	(c)
(B)	(b)	(c)	(a)	(d)
(C)	(c)	(d)	(a)	(b)
(D)	(b)	(c)	(d)	(a)

15. The value of $(2x+1)+(x+2)$ in $Z_3[x]$ is

(A) 1

(B) $x+2$

(C) 0

(D) $2x+1$

ALT

15. If X_1 and X_2 are two independent normal variates with means μ_1 and μ_2; and variances ${\sigma_1}^2$ and ${\sigma_2}^2$ respectively then $X_1 - X_2$ is also a normal variate with

(A) mean $\mu_1 - \mu_2$ and variance ${\sigma_1}^2 - {\sigma_2}^2$

(B) mean μ_1 / μ_2 and variance ${\sigma_1}^2 - {\sigma_2}^2$

(C) mean $\mu_1 + \mu_2$ and variance ${\sigma_1}^2 + {\sigma_2}^2$

(D) mean $\mu_2 - \mu_1$ and variance ${\sigma_2}^2 - {\sigma_1}^2$

16. Consider the following statements :

(i) $Q[x] / \langle x^2+1 \rangle$ is a field

(ii) Every field of characteristic p contains a subfield isomorphic to Q

(iii) Q is its own prime subfield

(iv) The prime subfield of C is R

Find the correct answer from the codes below :

Codes :	(i)	(ii)	(iii)	(iv)
(A)	False	True	True	False
(B)	True	False	True	False
(C)	True	True	True	False
(D)	False	True	True	True

ALT

16. The random variables X and Y have the joint *pdf*

$f(x, y) = 24xy,\ x > 0,\ y > 0,\ x+y \le 1$
$= 0,$ elsewhere

Then the variables $U = X+Y$ and $V = X/Y$ are

(A) independent beta distribution of first kind and gamma distribution respectively.

(B) independent beta distribution of first kind and beta distribution of 2nd kind respectively.

(C) both beta distribution of 2nd kind.

(D) independent gamma distributions.

Choose the correct answer from the above list.

17. Let $p = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $q = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ belong to the multiplicative group G of all (2×2) non-singular matrices over $\mathbb{R}$. Then in G

(A) orders of p, q and $p.q$ are all finite.

(B) orders of p, q are finite but order of $p.q$ is infinite

(C) each element is of finite order

(D) order of p = order of q

ALT

17. For a positively skewed distribution identify the correct options :

(A) mean $\leq$ median $\leq$ mode

(B) mean $\geq$ median $\geq$ mode

(C) mean of the distribution does not exist

(D) mean of the distribution is also positive

18. Choose the correct matching for the transformations T_1, T_2, T_3 (from $\mathbb{R}^2$ to $\mathbb{R}^3$) as defined in List I with the statements given in List II :

List I	List II
(a) $T_1(x, y) = (x, x, 0)$	(i) linear transformation of rank 2
(b) $T_2(x, y) = (x, x+y, y)$	(ii) not a linear transformation
(c) $T_3(x, y) = (x, x+1, y)$	(iii) linear transformation of rank 1

Codes :

	(a)	(b)	(c)
(A)	(iii)	(i)	(ii)
(B)	(i)	(ii)	(iii)
(C)	(iii)	(ii)	(i)
(D)	(i)	(iii)	(ii)

ALT

18. Let the sequence $\{T_n\}$ be such that, for each $\theta \in (H)$, the expectation and variance of T_n exist and $E_\theta(T_n) \to \gamma(\theta)$ and $Var_\theta(T_n) \to 0$ as $n \to \infty$. Then

(A) T is a consistent estimator of $\gamma(\theta)$

(B) T is a minimum chi-square estimator of $\gamma(\theta)$

(C) T is a sufficient estimator of $\gamma(\theta)$

(D) None of the above

19. Consider the following statements regarding the differential equation

$$\left|\frac{dy}{dx}\right|+|y|=0, \quad 0<x<1$$

satisfying $y(0)=1$:

1. It is a linear differential equation
2. It has a unique solution

Which of the statements given above is/are correct?

(A) 1 only

(B) 2 only

(C) neither 1 nor 2

(D) both 1 and 2

ALT

19. Let $X_{1j}\,(j=1,2\ldots n_1)$ be a random sample from a normal distribution with mean μ_1, $X_{2j}\,(j=1,2\ldots n_2)$ be a random sample from a second normal distribution with mean μ_2 . . ., and $X_{kj}\,(j=1,2,\ldots n_k)$ be a random sample from a kth normal distribution with mean μ_k. The k normal distributions is supposed to be homoscedastic with equal variance σ^2. In this case the likelihood function is

(A) $\dfrac{1}{(2\pi\sigma^2)_{n=\sum_{i=1}^{k} n_i}} exp\left[-\sum_i\sum_j (x_{ij}-\mu_i)^2/2\sigma^2\right]$

(B) $\dfrac{1}{(2\pi\sigma^2)^{n/2}} exp\left[-\sum_i (x_{ij}-\mu_i)^2/2\sigma^2\right]$

(C) $\dfrac{1}{(2\pi\sigma^2)^{n/2}} exp\left[-\sum_i\sum_j (x_{ij}-\mu_i)^2/2\sigma^2\right]$

(D) None of the above

20. If $u_1=\dfrac{\partial u}{\partial x}$, $u_2=\dfrac{\partial u}{\partial y}$, $u_3=\dfrac{\partial u}{\partial z}$ then the equations $f(x;y,z,u_1,u_2,u_3)=0$ and $g(x,y,z,u_1,u_2,u_3)=0$ are compatible if

(A) $\dfrac{\partial(f,g)}{\partial(x,u_1)}+\dfrac{\partial(f,g)}{\partial(y,u_2)}+\dfrac{\partial(f,g)}{\partial(z,u_3)}=1$

(B) $\dfrac{\partial(f,g)}{\partial(x,u_1)}+\dfrac{\partial(f,g)}{\partial(y,u_2)}+\dfrac{\partial(f,g)}{\partial(z,u_3)}=0$

(C) $\dfrac{\partial(x,u_1)}{\partial(f,g)}+\dfrac{\partial(y,u_2)}{\partial(f,g)}+\dfrac{\partial(z,u_3)}{\partial(f,g)}=1$

(D) $\dfrac{\partial(x,u_1)}{\partial(f,g)}+\dfrac{\partial(y,u_2)}{\partial(f,g)}+\dfrac{\partial(z,u_3)}{\partial(f,g)}=0$

ALT

20. In a 2^3 factorial experiment we arrange the treatments

(A) Only in Randomised block design (RBD)

(B) Either in Randomised block design (RBD) or Latin square design (LSD)

(C) Only in LSD

(D) Only in CRD

21. If $y = x$ is a solution of $x^2y'' + xy' - y = 0$, then the second linearly independent solution of the above equation is

(A) $1/x$

(B) x^2

(C) x^{-2}

(D) x^n

ALT

21. In a Latin square design each treatment occurs once and only once in

(A) each row and each column

(B) each row

(C) in column

(D) one row

22. Consider the following two statements — Assertion (Ass) and Reason (R) and find out the correct answer from the codes given below :

Assertion (Ass) : The equation $\frac{d^2y}{dx^2} = 5\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}$ does not represent an ordinary differential equation.

Reason (R) : The reason is that the degree of the equation is a fraction.

Codes :

(A) Both (Ass) and (R) are false

(B) (Ass) is false but (R) is true

(C) (Ass) is true but (R) is false

(D) Both (Ass) and (R) are true

ALT

22. Basic principles of design of experiment are :

(A) Replication, Randomisation and Local control

(B) Randomisation, Replication

(C) Randomisation and Local control

(D) Only Replication

23. Consider the following two statements Assertion (Ass) and Reason (R) and find out the correct answer from the codes given below :

Assertion (Ass) : If $p(x)$ is a real polynomial and $k \in \mathbb{R}$ then between any two real roots of $p(x) = 0$ there is a root of $p'(x) + kp(x) = 0$.

Reason (R) : The polynomial equation $p'(x) + kp(x) = 0$ must have a root.

Codes :

(A) Both (Ass) and (R) are true and (R) is the correct explanation of (Ass)

(B) Both (Ass) and (R) are true but (R) is not the correct explanation of (Ass)

(C) (Ass) is true, but (R) is false

(D) (Ass) is false, but (R) is true

ALT

23. Suppose we have the following values of the variables X and Y

X :	1	2	3	4	5
Y :	5	6	7	8	9

Then the regression coefficient of Y on X is

(A) 2

(B) 1

(C) 0·5

(D) 0·8

24. Consider the following two statements Assertion (Ass) and Reason (R) and find out the correct answer from the codes given below :

Assertion (Ass) : The function $f(z)=\sqrt{|xy|}$ is not analytic at the origin

Reason (R) : Cauchy-Riemann equations (C-R equations) are not satisfied at the origin.

Codes :

(A) (Ass) is false but (R) is not the correct explanation of (Ass)

(B) (Ass) is true and (R) is the correct explanation of (Ass)

(C) (Ass) is false, but (R) is true

(D) (Ass) is true, but (R) is false

ALT

24. Graphical representation of data achieves (identify the correct alternative) :

(A) Quantitative measure of central tendency

(B) Quantitative measure of dispersion

(C) Quantitative measures of both central tendency and dispersion

(D) None of the above.

25. The number of homomorphisms of Q into itself is

(A) 1

(B) 3

(C) infinitely many

(D) 2

ALT

25. Let Ω be the sample space with respect to a random experiment. Also, let A be an event and ϕ is the null set or impossible event. Then, the smallest field containing A is given by (Choose the correct one)

(A) $\{A, A^c\}$

(B) $\{A, A^c, \phi\}$

(C) $\{A, A^c, \Omega\}$

(D) $\{A, A^c, \phi, \Omega\}$

26. Consider the following two statements Assertion (Ass) and Reason (R) and find out the correct answer from the codes given below :

Assertion (Ass) : The dimension of the vector space $\mathcal{P}_n$ of all real polynomials in x of degree $< n$ is n.

Reason (R) : The set of polynomials $\{1, x, x^2, \ldots, x^{n-1}\}$ is a basis of $\mathcal{P}_n$

Codes :

(A) (Ass) is false, but (R) is true

(B) (Ass) is false, since (R) is not true

(C) Both (Ass) and (R) are true and (R) is the correct explanation of (Ass)

(D) Both (Ass) and (R) are true but (R) is not the correct explanation of (Ass)

ALT

26. Let X follow $b(n, \theta)$ distribution, $0 < \theta < 1$. Also, let the correlation coefficient between X and $n - X$ be r.

Then choose the correct answer from the following :

(A) $r = 0 \cdot 5$

(B) $r = +1$

(C) $r = -1$

(D) $r = 0$

27. In a two person zero sum game, which of the following is correct ?

(A) The saddle point will always exist

(B) A mixed strategy will never exist

(C) A mixed strategy can never be implemented

(D) The pure strategy is a particular case of mixed strategy

ALT

27. Let X be a random variable with the probability density function (pdf) given by

$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, \quad -\infty < x < \infty$$

Then consider the following statements :

(i) $E(|X|)$ is not finite

(ii) Median of X is zero

(iii) Moment generating function of X does not exist

(iv) The distribution is symmetrical about its mean.

Choose the correct relations from the following true-false table :

Codes :	(i)	(ii)	(iii)	(iv)
(A)	False	True	False	True
(B)	True	False	True	False
(C)	False	False	False	True
(D)	True	True	True	False

8. The set $S = \{(x_1, x_2) : x_2^2 \leq x_1\}$ has

(A) 4 vertices

(B) no vertex

(C) finite number of vertices

(D) infinite number of vertices

ALT

28. The maximum likelihood estimator (MLE) has become a widely accepted type of estimator due to the following reason.
Choose the correct one :

(A) Whatever the form of the distribution, the likelihood equation always gives a simple solution

(B) The distributions of MLEs are easy to work out irrespective of sample size

(C) MLEs are always unbiased

(D) MLEs have nice large sample properties

29. Match the following functions with their differentiability and select the correct match using codes below :

Functions	Differentiability
1. $f(x) = x\sin\frac{1}{x}$, if $x \neq 0$; and $f(0) = 0$	(i) is differentiable at $x = 0$
2. $I(x) = x^2\sin\frac{1}{x}$, if $x \neq 0$; and $I(0) = 0$	(ii) is differentiable at $x = 0$
3. $h(x) = x^2\cos\frac{1}{x}$, if $x \neq 0$; and $h(0) = 0$	(iii) is not differentiable at $x = 0$
4. $\phi(x) = x\cos\frac{1}{x}$, if $x \neq 0$; and $\phi(0) = 0$	(iv) is not differentiable at $x = 0$

Codes :

	1	2	3	4
(A)	(i)	(iii)	(iv)	(ii)
(B)	(iii)	(ii)	(i)	(iv)
(C)	(ii)	(iv)	(i)	(iii)
(D)	(iv)	(i)	(ii)	(iii)

ALT

29. In executing the analysis of variance of a given data set, certain assumptions are made. Identify the assumption from below which is not appropriate in this type of analysis :

(A) The data follow a normal distribution

(B) The errors are correlated

(C) The error variance is homoscedastic

(D) The different effects are additive

30. The following are given statements :
The differential equation

(a) $\frac{dy}{dx} = \sqrt{y}$ with initial condition $y(0) = 0$ has infinitely many solution

(b) $\frac{dy}{dx} = \frac{y}{\sqrt{x}}$, $y(1) = 2$ has unique solution defined in some sufficiently small interval about $x_0 = 1$

(c) $x\frac{dy}{dx} = y$, $y(0) = 0$ has uncountably many solutions

(d) $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0$, $|x| \leq \frac{1}{\sqrt{2}}$ has finitely many linearly independent solutions.

Use the truth table given below to choose the correct answer.

Codes : (a) (b) (c) (d)

(A) False False False True

(B) True True True False

(C) True False True True

(D) False False True False

30. Consider the following statements :

(a) In an ordinary sign test we consider both sign and magnitude of the differences.

(b) Mann-Whitney U-test is one of the most widely used one sample non-parametric test

(c) The Kolmogorov-Smirnov statistics are not distribution free

(d) In the Wilcoxon signed-rank test we assume that the population is continuous and symmetric

Use the truth table given below to choose the correct answer :

Codes : (a) (b) (c) (d)

(A) True False False False

(B) False False False True

(C) False False True False

(D) False True False False

31. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ then the distribution of $\sum_{i=1}^{n} \frac{X_i^2}{\sigma^2}$ is a Central χ^2 with n *d.f.* if (identify the correct one) :

(A) $\mu = 0$

(B) $\mu < 0$

(C) $\mu > 0$

(D) σ^2 is finite

ALT

31. Let V be a real vector space of all mappings from $\mathbb{R}$ to $\mathbb{R}$ Let $V_1 = \{f \in V : f(-x) = f(x)\}$ and $V_2 = \{f \in V : f(-x) = -f(x)\}$. Then which one of the following is correct ?

(A) Both V_1 and V_2 are subspaces of V

(B) V_1 is a subspace of V, but V_2 is not a subspace of V

(C) V_2 is a subspace of V, but V_1 is not a subspace of V

(D) Neither V_1 nor V_2 is a subspace of V

32. Which of the following is correct ?

(A) A customer's behaviour to keep on switching from one queue to another is called jockeying.

(B) Arrival of patients with appointment to a doctor can be described as arrival by Poisson process.

(C) Constant service time is a special case of Poisson service time.

(D) Arrival of passengers from train at a particular station is an example of individual arrival of customers.

33. Which of the following is not correct ?

(A) Queuing theory deals with situations where customers arrive, wait for a service, get the service and leave the system.

(B) Customers, in queuing theory, might include humans, machines, ships, letters and so on.

(C) A queue refers to physical presence of the customers waiting to be served.

(D) A study of queuing theory helps the manager to establish an optimum level of service.

34. Let S be the set of all feasible solutions of an L.P.P, $Ax = b$, $x \geq 0$ and $x^* \in S$ minimize the objective function $Z = cx$. Suppose that x^* also maximizes the function Z^* over S. Then

(A) $Z^* = (-c)x$

(B) $Z^* = cx$

(C) $Z^* > (-c)x$

(D) $Z^* < (-c)x$

35. The minimum number of lines covering all zeros in a reduced cost matrix of order n can be

(A) at most n

(B) at least n

(C) $n+1$

(D) $n+2$

36. In a balanced transportation problem if u_i's and v_j's are the dual variables associated with rows and columns, then

(A) $u_i \geq 0$

(B) $v_j \geq 0$

(C) $u_i, v_j \geq 0$

(D) u_i, v_j unrestricted

37. Let us consider the following statements :

(a) $arg(z) - arg(-z) = \pi$, if $arg(z) > 0$

(b) $arg(z) - arg(-z) = -\pi$, if $arg(z) > 0$

(c) $arg(z) - arg(-z) = -\pi$, if $arg(z) < 0$

(d) $arg(z) - arg(-z) = \pi$, if $arg(z) < 0$

Use the truth table given below to choose the correct answer.

Codes :	(a)	(b)	(c)	(d)
(A)	False	True	False	True
(B)	False	True	True	False
(C)	True	False	True	False
(D)	True	False	False	True

ALT

37. Let V_{Ney}, V_{Prop} and V_{ran} be the variances, respectively, of Neyman allocation, proportional allocation and mean per unit under simple random sampling. Then choose the correct relationship from the following alternatives :

(A) $V_{ran} \leq V_{Prop} \leq V_{Ney}$

(B) $V_{Ney} \leq V_{ran} \leq V_{Prop}$

(C) $V_{Ney} \leq V_{Prop} \leq V_{ran}$

(D) $V_{ran} \leq V_{Ney} \leq V_{Prop}$

38. Let $\{a_n\}=\{n^2\}$ and $\{b_n\}=\{-n^2\}$ be two real sequences, then

(A) the sequence $\left\{a_n/b_n\right\}$ is convergent but the sequences $\{a_n\}$ and $\{b_n\}$ are divergent.

(B) the sequence $\left\{a_n/b_n\right\}$ is divergent but the sequences $\{a_n\}$ and $\{b_n\}$ are convergent.

(C) the sequences $\left\{a_n/b_n\right\}$ and $\{a_n\}$ are convergent but the sequence $\{b_n\}$ is divergent.

(D) None of the sequences $\left\{a_n/b_n\right\}$, $\{a_n\}$ and $\{b_n\}$ is convergent.

38. In a simple random sampling with replacement (SRSWR) of size n, let ν be the number of distinct units in the sample. Also, let $\bar{y}_n$ and $\bar{y}_\nu$ be the means based on all observations and distinct observations respectively, which are proposed to be the estimates of the population mean $\bar{Y}$. Then identify the wrong statement from the following :

(A) $Var(\bar{y}_n) \leq Var(\bar{y}_\nu)$

(B) $Var(\bar{y}_n) \geq Var(\bar{y}_\nu)$

(C) Both $\bar{y}_n$ and $\bar{y}_\nu$, based on n and ν observations respectively, are unbiased for $\bar{Y}$.

(D) The quantity ν is a random variable.

39. Let f be a meromorphic function. Then its

(A) zeros are isolated points but poles are not so

(B) poles are isolated points but zeros are not so

(C) both poles and zeros are isolated points

(D) neither zeros nor poles are isolated points.

ALT

39. Considering Assertion (Ass) and Reason (R), choose the correct answer from the given codes :

Assertion (Ass) : In a 2^3 factorial experiment 7 degrees of freedom for treatment can be broken up into seven independent components of one degrees of freedom each.

Reason (R) : Because all the main effects and interaction effects are mutually orthogonal contrasts.

Codes :

(A) (Ass) is false, but (R) is true

(B) (Ass) is true, but (R) is false

(C) Both (Ass) and (R) are true

(D) Both (Ass) and (R) are false

40. Choose the correct answer :

(A) The order of $\sigma = (4,5)(2,3,7)$ is 3 and $\tau = (1,4)(3,5,7,8)$ is 8

(B) The order of $\sigma = (4,5)(2,3,7)$ is 6 and $\tau = (1,4)(3,5,7,8)$ is 4

(C) The order of $\sigma = (4,5)(2,3,7)$ is 6 and $\tau = (1,4)(3,5,7,8)$ is 8

(D) The order of $\sigma = (4,5)(2,3,7)$ is 2 and $\tau = (1,4)(3,5,7,8)$ is 2

ALT

40. Sources of variation in a randomised block design (RBD) are :

(A) Block and Treatment

(B) Block, Treatment and Error

(C) Block and Error

(D) Treatment and Error.

GROUP-A
(PURE AND APPLIED MATHEMATICS)

This contains **sixty (60)** objective type questions, each question carrying **two (2)** marks. **All** questions are compulsory.

41. Consider the following Assertion (Ass) and Reason (R). Choose correct answer from the codes given below :

Assertion (Ass) : $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = [x]$ is integrable over $[1, 5]$.

Reason (R) : f has countable number of discontinuities in $[1, 5]$

Codes :

(A) (Ass) is true but (R) is false

(B) Both (Ass) and (R) are true

(C) Both (Ass) and (R) are false

(D) (Ass) is false but (R) is true

42. Which one of the following statements is true for the sequence of functions $f_n(x) = \frac{1}{n^2 + x^2}$, $n = 1, 2, \ldots$, $x \in \left[\frac{1}{2}, 1\right]$?

(A) The sequence is monotonic and has 0 as the limit for all $x \in \left[\frac{1}{2}, 1\right]$ as $n \to \infty$

(B) The sequence is not monotonic but has $f(x) = \frac{1}{x^2}$ as the limit as $n \to \infty$

(C) The sequence is monotonic and has $f(x) = \frac{1}{x^2}$ as the limit as $n \to \infty$

(D) The sequence is not monotonic but has 0 as the limit.

43. Let $f_n : [1, 2] \to [0, 1]$ be given by $f_n(x) = (2 - x)^n$ for all non-negative integers n. Let $f(x) = \lim_{n \to \infty} f_n(x)$ for $1 \le x \le 2$. Then which of the following is true?

(A) f is a continuous function on $[1, 2]$

(B) f_n converges uniformly to f on $[1, 2]$ as $n \to \infty$

(C) $\lim_{n \to \infty} \int_1^2 f_n(x)\,dx = \int_1^2 f(x)\,dx$

(D) For any $a \in (1, 2)$, we have $\lim_{n \to \infty} f_n'(a) \neq f'(a)$

44. Match List I and List II and then choose the correct match from the codes given below :

List I	List II
(a) The radius of convergence of the power series $\sum \left(1 + \frac{1}{n}\right)^{n^2} z^n$	(i) is 1
(b) The radius of convergence of the power series $\sum (\log n)^n z^n$	(ii) is 0
(c) The radius of convergence of the power series $\sum \frac{z^n}{n!}$	(iii) is ∞
(d) The radius of convergence $\sum_{n=1}^{\infty} \frac{1}{n!} z^{n^2}$	(iv) is $\frac{1}{e}$

Codes :

	(a)	(b)	(c)	(d)
(A)	(i)	(iii)	(iv)	(ii)
(B)	(ii)	(i)	(iv)	(iii)
(C)	(iv)	(ii)	(iii)	(i)
(D)	(iv)	(iii)	(ii)	(i)

45. Let $R = Z[i]$ and $\alpha = 1+i$. If $I = \langle 1+i \rangle$, the ideal generated by $1+i$ then

(A) I is a maximal ideal of R

(B) I is a Prime ideal of R

(C) $I = R$

(D) I is a maximal ideal and Prime ideal of R

46. The alternating group A_4 on 4 symbols has a normal subgroup of order

(A) 2

(B) 3

(C) 4

(D) 6

47. Let (X, d) be a complete metric space and $f : X \to X$ satisfies $d(f(x), f(y)) \leq \alpha d(x, y)$ for some α, $0 \leq \alpha < 1$ for all $x, y \in X$. Then

(A) f is a bounded function on X

(B) f need not be continuous on X

(C) $\{f(x_n)\}_{n=1}^{\infty}$ may not be a Cauchy sequence even through $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X

(D) $f(p) = p$ for some $p \in X$

48. In a metric space (X, d)

(A) every closed and bounded set is compact

(B) there exists a closed set A in X, which strictly contains the set $\{x \in X : d(x, A) = 0\}$, where $d(x, A) = U\{d(x, a) : a \in A\}$

(C) every infinite set A has a limit point in A

(D) every closed subset of a compact set is compact

49. Let B be a subset of $\mathbb{R}$ with $\mu^*(B) > 0$. Then

(A) there exists a non (Lebesgue) measurable subset B_1 of B such that $B_1 \subsetneq B$

(B) each subset of B is Lebesgue measurable

(C) each infinite subset of B is non-measurable

(D) B necessarily contains an open interval I which is not degenerated

50. Let R be a right Artinian ring then

(A) the radical is nilpotent

(B) the radical is the smallest nilpotent ideal

(C) the prime radical is the smallest nilpotent ideal

(D) The radical and the prime radical are the smallest nilpotent ideal

51. A group G of order 30 is :

(A) a simple group

(B) G has at least two subgroups of order 5

(C) G has only one subgroup of order 5 which is a normal subgroup of G

(D) G has only one subgroup of order 5 which is not normal in G.

52. The symmetric group S_n

(A) is not solvable for $n < 5$

(B) is not solvable for $n = 4$

(C) is not solvable for $n \geq 5$

(D) is not solvable for $n = 2, 3$

53. Which of the following is false?

(A) The space $L^1[0,1]$ is separable.

(B) The space l_1 is a Banach space.

(C) Every linear map from a normed linear space X to a finite dimensional normed linear space Y is continuous.

(D) Let X be a normal linear space which is not a Banach space. Then X cannot be reflexive.

54. Which of the following is true?

(A) There exists a countably infinite dimensional Banach space.

(B) There exists an infinite dimensional normed linear space X such that each linear functional on X is continuous.

(C) Any infinite dimensional Hilbert space X over $\mathbb{C}$ is isomorphic to l_2.

(D) A Hilbert space X is separable iff there exists a complete countable orthonormal set in X.

55. Which of the following is not true?

(A) All norms on a vector space X are equivalent provided X is finite dimensional.

(B) Let X be the space of polynomials in one Variable. For $p \in X$, $p(t) = a_0 + a_1 t + \ldots + a_n t^n$. Let $\|p\| = \sup\{|p(t)| : 0 \leq t \leq 1\}$ and $\|p\|_1 = |a_0| + |a_1| + \ldots + |a_n|$. Then X is not complete with respect to the above norm.

(C) The statement "the dual space of non-empty normed linear space is non-empty" follows from 'Hahn-Banach Theorem'.

(D) Consider the Banach space $C[0, \pi]$ with the supremum norm. The norm of the linear functional $l : C[0,\pi] \to R$ given by $l(f) = \int_0^{\pi} f(x)\sin^2 x\,dx$ is π.

56. Which of the following sets is compact in $\mathbb{R}^2$ with usual topology?

(A) $\{(x,y) : xy = 1\}$

(B) $\{(\sin x, \cos x) : x \in \mathbb{R}\}$

(C) $\{(x, x^2) : x \in (0,1)\}$

(D) $\{(x, y) \in Q^2 : x^2 + y^2 = 1\}$

57. For the product topological space $X = [0,1]^{[0,1]}$, which of the following is false?

(A) X is first countable

(B) X is compact and T_2

(C) X is connected

(D) X is a bijective continuous image of a discrete space Y with $|Y| = 2^c$

58. A person wants to leave station B. There are three routes from station B to A and four routes from B to C. The number of ways the person leave the station B is

(A) 3

(B) 2

(C) 7

(D) 4

59. Let L be a modular lattice. For $a, b, c \in L$ with $a \geq b$

(A) $a \wedge (b \vee c) = b \vee (a \wedge c)$

(B) $a \wedge (b \vee c) = a \vee (b \wedge c)$

(C) $a \vee (b \wedge c) = b \wedge (a \vee c)$

(D) $a \wedge (b \vee c) = b \wedge (a \vee c)$

60. Consider the following Assertion (Ass) and Reason (R); choose the correct answer from the codes given below :

Assertion (Ass) : Peterson graph is non-planar

Reason (R) : Peterson graph contains a subgraph homeomorphic to $K_{3,3}$

Codes :

(A) (Ass) is true but (R) is false

(B) (Ass) is false but (R) is true

(C) Both (Ass) and (R) are true

(D) Both (Ass) and (R) are false

61. Consider the following statements :

(i) The graph K_3 is Eulerian.

(ii) The graph $K_{2,3}$ is Eulerian.

(iii) The graph $K_{4,4}$ is Eulerian.

(iv) The graph $K_{4,5}$ is Eulerian.

Find the correct answer from the following codes :

Codes : (i) (ii) (iii) (iv)

(A) True False True False

(B) True True False False

(C) False False True True

(D) Flase True False True

62. Which of the following is true ?

(A) The singular solution of the differential equation $y = \frac{2}{3}x\frac{dy}{dx} - \frac{2}{3x}\left(\frac{dy}{dx}\right)^2, x>0$ is $y = \frac{1}{6}x^3$

(B) The order of the differential equation $\left(\frac{dy}{dx}\right)^4 \frac{d^2y}{dx^2} + \sqrt{\frac{d^3y}{dx^3}} + \sqrt{\left(\frac{dy}{dx}\right)^5} = 0$ is 5

(C) If y_1 and y_2 be two solutions of the differential equation $\frac{d^2y}{dx^2} + 2y = f(x)$, then $y_1 + y_2$ is also a solution of $\frac{d^2y}{dx^2} + 4y = 4f(x)$

(D) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ is $\sin\left(\frac{y}{x}\right) = Cx$

63. Which of the following is not true ?

(A) The equation $x^2(y-1)z_{xx} - x(y^2-1)z_{xy} + y(y-1)z_{yy} + z_x = 0$ is hyperbolic in the entire xy-plane except along y-axis.

(B) The partial differential equation

$$y^2u_{xx} - 2xy\,u_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$$

can be classified as hyperbolic.

(C) The partial differential equation $(1+x^2)u_{xx} + (1+y^2)u_{yy} + xu_x + yu_y = 0$ can be classified as elliptic, if $x<0$, $y<0$; hyperbolic if $x>0$, $y>0$ and parabolic if $x=0$, $y=0$.

(D) The partial differential equation $z_{xx} - x^2y\,z_{yy} = 0, y>0, x>0$ can be classified as hyperbolic.

64. The solution of the differential equation

$$x^2y\frac{d^2y}{dx^2} + \left(x\frac{dy}{dx} - y\right)^2 = 0$$

is given by

(A) $y^2 = C_1x$

(B) $y^2 = C_1x^2 + C_2x^3$

(C) $y^2 = C_2x^2 - 2C_1x$

(D) $y^2 = C_1 + C_2x$

65. All eigenvalues and eigenfunctions of the Sturm-Liouville problem $y'' + \lambda y = 0$; $y(0) = 0$, $y'\left(\frac{\pi}{2}\right) = 0$ are given by

(A) $(2m-1)^2$, $\sin(2m-1)x$

(B) $(m\pi)^2$, $\cos(2m-1)x$

(C) $(2m-1)$, $\sin(2m-1)x$

(D) $2m\pi$, $\cos 2mx$

66. Three-dimensional Laplace's equation in cylindrical polar coordinates (ρ, ϕ, z) is given by

(A) $\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial u}{\partial \rho} + \frac{1}{\rho}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(B) $\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(C) $\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{\partial^2 u}{\partial \phi^2} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial z^2} = 0$

(D) $\frac{1}{\rho}\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial u}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$

67. Let p be an odd prime. Then the quadric congruence

$$x^2 \equiv 2 \pmod{p}$$

has a solution if

(A) $p \equiv 1 \pmod 8$, $p \equiv 3 \pmod 8$

(B) $p \equiv 3 \pmod 8$, $p \equiv 5 \pmod 8$

(C) $p \equiv 1 \pmod 8$, $p \equiv 7 \pmod 8$

(D) $p \equiv 3 \pmod 8$, $p \equiv 7 \pmod 8$

68. The remainder when $111^{333} + 333^{111}$ is divided by 7 is

(A) 6

(B) 4

(C) 2

(D) 0

69. The Diophantine equation $x^2 - 4y^2 = 2$ has

(A) two solutions

(B) four solutions

(C) unique solution

(D) no solution

70. Consider the quadratic congruences

(a) $x^2 \equiv 71 \pmod{73}$

(b) $x^2 \equiv 19 \pmod{23}$

then

(A) (a) is solvable but (b) is not solvable

(B) (b)is solvable but (a) is not solvable

(C) both (a) and (b) are solvable

(D) both (a) and (b) are not solvable

71. If the kinetic energy is conserved for a system then out of all possible paths between two points the particle moves along the path for which

(A) path is shortest

(B) the transit time is extremum

(C) both (A) and (B)

(D) the transit time is minimum

72. The Lagrange's equation of motion for a simple pendulum can be represented by

(A) $\ddot{x} + xk = 0$

(B) $\ddot{x} + k \sin x = 0$

(C) $\ddot{x} + \frac{k}{m} x = 0$

(D) $\ddot{x} + \frac{k}{m} \sin x = 0$

73. If $[\phi, \psi]$ be the Poisson bracket then $\frac{\partial}{\partial t}[\phi, \psi]$ is given by

(A) $\left[\frac{\partial \phi}{\partial t}, \frac{\partial \psi}{\partial t}\right]$

(B) $\left[\frac{\partial \phi}{\partial t}, \psi\right] + \left[\phi, \frac{\partial \psi}{\partial t}\right]$

(C) $\left[\frac{\partial \phi}{\partial t}, 1\right] + \left[1, \frac{\partial \psi}{\partial t}\right]$

(D) $\left[\frac{\partial \phi}{\partial t}, \frac{\partial \psi}{\partial t}\right] + \left[\frac{\partial \psi}{\partial t}, \frac{\partial \phi}{\partial t}\right]$

74. The first invariant of the Lagrangian strain tensor may be expressed in terms of the principal stretches as

(A) $\left[\left(\Lambda^2_{(\hat{e}_1)} - 1\right) + \left(\Lambda^2_{(\hat{e}_2)} - 1\right) + \left(\Lambda^2_{(\hat{e}_3)} - 1\right)\right]^{1/2}$

(B) $\left[\left(\Lambda^2_{(\hat{e}_1)} - 1\right) + \left(\Lambda^2_{(\hat{e}_2)} - 1\right) + \left(\Lambda^2_{(\hat{e}_3)} - 1\right)\right]/2$

(C) $\left[\left(\Lambda^2_{(\hat{e}_1)} - 1\right)\left(\Lambda^2_{(\hat{e}_2)} - 1\right)\left(\Lambda^2_{(\hat{e}_3)} - 1\right)\right]^{1/2}$

(D) $\left[\left(\Lambda^2_{(\hat{e}_1)} - 1\right)\left(\Lambda^2_{(\hat{e}_2)} - 1\right)\left(\Lambda^2_{(\hat{e}_3)} - 1\right)\right]/2$

75. If the constants Young's modulus and Poisson's ratio are represented by E and ν respectively then for a state of pure shear, the shear modulus G is expressed by the relation

(A) $\frac{E}{1+\nu}$

(B) $\frac{E}{2(1+\nu)}$

(C) $\frac{E}{1-\nu}$

(D) $\frac{E}{2(1-\nu)}$

76. If $\sigma_{ij} = -p\delta_{ij}$ where p is a positive constant then the stress power $D_{ij}\sigma_{ij}$ is expressed in the form

(A) $p\frac{d\rho}{dt}$

(B) $\rho\frac{dp}{dt}$

(C) $\frac{\rho}{p}\frac{d\rho}{dt}$

(D) $\frac{p}{\rho}\frac{d\rho}{dt}$

77. The Cauchy's stress quadric at a point P in a continuum for a state of stress

$$(\tau_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

represents

(A) Ellipsoid

(B) Hyperboloid

(C) Paraboloid

(D) None of these

78. The stress tensor at a point P in a continuum is given by

$$(\tau_{ij}) = \begin{pmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{pmatrix}$$

where a, b, c are constants and σ is some stress value. If the stress vector on the octahedral plane with normal

$$\bar{v} = \frac{1}{\sqrt{3}}\bar{e}_1 + \frac{1}{\sqrt{3}}\bar{e}_2 + \frac{1}{\sqrt{3}}\bar{e}_3$$

vanishes then the values of a, b and c are

(A) $a = \frac{1}{2}$, $b = \frac{1}{2}$, $c = \frac{1}{2}$

(B) $a = -\frac{1}{2}$, $b = -\frac{1}{2}$, $c = -\frac{1}{2}$

(C) $a = \frac{1}{2}$, $b = -\frac{1}{2}$, $c = \frac{1}{2}$

(D) $a = -\frac{1}{2}$, $b = \frac{1}{2}$, $c = -\frac{1}{2}$

79. "A body moving with uniform velocity through an infinite liquid, otherwise at rest, will experience no resistance at all" is the statement of

(A) Euler's paradox

(B) D'Alembert's paradox

(C) Whitehead paradox

(D) Stokes's paradox

80. Consider the following two statements labelled as Assertion (Ass) and Reason (R) :

Assertion (Ass) : Slow motion can be developed in a viscous liquid

Reason (R) : When the Reynolds number is large

Select the correct answer from the following codes :

(A) Both (Ass) and (R) are true

(B) (Ass) is true but (R) is false

(C) (Ass) is false but (R) is true

(D) Both (Ass) and (R) are false

81. In the motion of a fluid, if the flow is directed radially inwards to a point from all directions in symmetrical manner then the point is called

(A) source

(B) sink

(C) doublet

(D) triplet

82. A stream in a horizontal pipe, after passing a contraction in the pipe in which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is B. If a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth h below the pipe, s being the delivery per second then h is given by

(A) $\frac{s^2}{2g}\left(\frac{1}{A^2}+\frac{1}{B^2}\right)$

(B) $\frac{2g}{s^2}\left(\frac{1}{A^2}+\frac{1}{B^2}\right)$

(C) $\frac{s^2}{2g}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$

(D) $\frac{2g}{s^2}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$

83. The extremals of the functional

$$I[y(x)]=\int_{x_0}^{x_1}\left(2xy+y'''^2\right)dx$$

lies on

(A) five parameter family of curves

(B) six parameter family of curves

(C) seven parameter family of curves

(D) four parameter family of curves

84. The functional

$$I[y(x)] = \int_{-l}^{l} \left(\frac{1}{2} \mu y''^2 + \lambda y \right) dx$$

subject to $y(-l) = 0$, $y'(-l) = 0$, $y(l) = 0$, $y'(l) = 0$ can be extremized by

(A) $D^4 y + \frac{\lambda}{\mu} y = 0$

(B) $D^4 y + \frac{\lambda}{\mu} = 0$

(C) $D^4 y - \frac{\lambda}{\mu} y = 0$

(D) $D^4 y - \frac{\lambda}{\mu} = 0$

85. The curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis gives a minimum surface area called

(A) cycloid

(B) catenary

(C) great circle

(D) family of circles

86. Consider the variational problem

$$I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\, dx$$ with $y(x_1) = y_1$ and the other boundary point moves on $y = \phi(x)$. Then the transversality condition for the moving boundary reduces to

(A) $\left[F + (\phi - y) F_{y'} \right]_{x=x_2} = 0$

(B) $\left[F - (\phi' - y') F_{y'} \right]_{x=x_2} = 0$

(C) $\left[F + (\phi' - y') F_{y'} \right]_{x=x_2} = 0$

(D) $\left[F_y + (\phi' - y') F_{y'} \right]_{x=x_2} = 0$

87. The real eigenvalues of th following integral equation

$$\phi(x) = \lambda \int_0^1 \left(\sqrt{x}\, t - \sqrt{t}\, x \right) dt$$

is

(A) $\sqrt{300}$

(B) $-\sqrt{300}$

(C) $\pm\sqrt{150}$

(D) does not exist

88. The integral equation

$$\int_0^x e^{x-t}\,\phi(t)\,dt = x$$

has a solution

(A) $\phi(x) = 1 + x$

(B) $\phi(x) = 1 - x$

(C) $\phi(x) = x$

(D) $\phi(x) = 1$

89. For a system of equations

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

a sufficient condition for convergence of the Gauss-Seidel iteration process

(A) $|a_{12}\,a_{21}| = |a_{11}\,a_{22}|$

(B) $|a_{12}\,a_{21}| > |a_{11}\,a_{22}|$

(C) $|a_{12}\,a_{21}| < |a_{11}\,a_{22}|$

(D) $|a_{11}| + |a_{22}| < |a_{12}| + |a_{22}|$

90. Let f be continuous map from the interval $[0, 1]$ into itself and consider its iteration $x_{n+1} = f(x_n)$. Which of the following will not yield a fixed point for f?

(A) $f(x) = \frac{3x^2}{4}$

(B) $f(x) = \frac{x^2}{8}$

(C) $f(x) = \frac{x^2}{16}$

(D) $f(x) = \frac{x^2}{32}$

91. The equation $e^x - 4x^2 = 0$ has a root between 4 and 5. Fixed point iteration with iteration function $\frac{1}{2}e^{\frac{x}{2}}$

(A) diverges

(B) converges

(C) oscillates

(D) converges monotonically

92. Consider the degree of precision for numerical integration given by different rules :

Rule	Degree of precision
(a) Trapezoidal rule	(i) 3
(b) Simpson's one-third rule	(ii) 1
(c) Simpson's three-eighth rule	(iii) 2
(d) Weddle's rule	(iv) 7

Find the correct match from the following matching table :

Codes :	(a)	(b)	(c)	(d)
(A)	(i)	(iv)	(ii)	(iii)
(B)	(iv)	(ii)	(iii)	(i)
(C)	(iii)	(i)	(iv)	(ii)
(D)	(ii)	(iii)	(i)	(iv)

93. Match the following functions and their proper Laplace transforms :

Functions	Laplace transforms
(a) e^{at}	(i) $\frac{\Gamma(n+1)}{s^{n+1}}$; $s>0, n>-1$
(b) $\sin at$	(ii) $\frac{s}{s^2-a^2}$, $s>\lvert a\rvert$
(c) $\cosh at$	(iii) $\frac{1}{s-a}$, $s>a$
(d) t^n for $n>-1$	(iv) $\frac{a}{s^2+a^2}$, $s>0$

where $\mathcal{L}\{f(t)\}=F(s)$

Choose the correct match from the following codes :

Codes :	(a)	(b)	(c)	(d)
(A)	(i)	(ii)	(iii)	(iv)
(B)	(ii)	(iii)	(iv)	(i)
(C)	(iii)	(iv)	(ii)	(i)
(D)	(iv)	(i)	(iii)	(ii)

94. The Fourier transform of the function $f(x)$ defined by

$$f(x)=1, \quad |x|<a$$
$$=0, \quad |x|>a$$

is $\frac{2}{s}\sin sa$ for $s\neq 0$, then the value of $\int_0^\infty \frac{\sin s}{s}\,ds$ is given by

(A) 1

(B) π

(C) $\pi/2$

(D) $\pi/3$

95. The Fourier cosine transform of the function e^{-x^2} is given by

(A) $\frac{\pi}{2}e^{-p^2/4}$

(B) $\frac{\sqrt{\pi}}{2}e^{-p^2/4}$

(C) $\frac{\pi}{2}e^{p^2/4}$

(D) $\frac{\sqrt{\pi}}{2}e^{p^2/4}$

96. Which of the following methods of solving a quadratic programming problem is based on modified simplex method ?

(A) Wolfe's method

(B) Beale's method

(C) Frank-Wolfe method

(D) Fletcher's method

97. Quadratic programming is concerned with the NLPP of optimizing the quadratic objective function subject to

(A) linear inequality constraints

(B) non-linear inequality constraints

(C) non-linear equality constraints

(D) no constraints

98. Which of the following is correct?

(A) Revised simplex method is applicable to only those LPP's which involve artificial variable.

(B) Revised simplex method is more economical when LPP is solved manually but is not very convenient when computer is used.

(C) Revised simplex method is not very convenient when an LPP is solved manually but is more economical on a computer.

(D) Revised simplex method does not permit the computation of large size LPP.

99. Let $(X, \mathcal{A}, \mu)$ be a measure space and let f be a real valued μ-integrable function. Let ν be a finite integral measure defined by

$$\nu(A) = \int_A f \, d\mu$$

then for each $A \in \mathcal{A}$

(A) $|\nu|(A) = \left| \int_A f \, d\mu \right|$

(B) $|\nu|(A) = \int_A |f| \, d\mu$

(C) $|\nu|(A) = \int_A f \, d\mu$

(D) $|\nu|(A) = -\int_A f \, d\mu$

100. Let L a lattice, for $a, b \in L$

(A) $a \wedge (a \vee b) = b$

(B) $a \wedge (a \vee b) = a \vee b$

(C) $a \wedge (a \vee b) = a$

(D) $a \wedge (a \vee b) = a \wedge b$

GROUP–B
(STATISTICS)

This contains **sixty (60)** objective type questions, each question carrying **two (2)** marks. **All** questions are compulsory.

41. Let $f_n : [0, 1] \to \mathbb{R}$ be defined by

$$f_n(x) = 2n, \quad \text{if } \frac{2}{2n} \le x \le \frac{1}{n}$$

$$= 0, \quad \text{otherwise}$$

Then the value of $\int_0^1 \lim_{n\to\infty} f_n d\mu$ and $\lim_{n\to\infty} \int_0^1 f_n d\mu$ (where μ is the Lebesgue measure on $\mathbb{R}$) are respectively.

(A) 0, 0

(B) 0, 1

(C) 1, 0

(D) 1, 1

42. A symmetric die is thrown 600 times. Choose the correct value of the lower bound for the probability of getting 80 to 120 sixes from the following :

(A) $\frac{5}{24}$

(B) $\frac{7}{24}$

(C) $\frac{19}{24}$

(D) $\frac{21}{24}$

43. Let $\{X_r : r \ge 1\}$ be independent Poisson variables with respective parameters $\{\lambda_r : r \ge 1\}$. Then $\sum_{r=1}^{\infty} X_r$ converges almost surely if (identify the correct one) :

(A) $\sum_{r=1}^{\infty} \lambda_r$ tends to ∞

(B) $\sum_{r=1}^{\infty} \lambda_r$ is finite

(C) $\sum_{r=1}^{\infty} \lambda_r$ tends to $-\infty$

(D) $\sum_{r=1}^{\infty} \lambda_r$ fluctuates

44. Consider a function g such that $g : \mathbb{R} \to \mathbb{R}$. Also, let g be continuous.

Examine the following statements :

Assertion (Ass) :

$$g(X_n) \xrightarrow{p} g(X)$$

Reason (R) : $X_n \xrightarrow{p} X$

Choose the correct answer from the codes below :

(A) Both (Ass) and (R) are true but (R) is not the correct explanation of (Ass).

(B) Both (Ass) and (R) are true and (R) is the correct explanation (Ass)

(C) (Ass) is false but (R) is true

(D) (Ass) is true but (R) is false

45. Let $X_1, X_2, ..., X_n, ...$ be independent random variables with mean zeros and let $S_n = X_1 + X_2 + ... + X_n$. Then, Kolmogorov's inequality states that

$P\left(\max_{1\le k\le n}|S_k| \le c\right), (c>0)$ is (identify the correct alternative)

(A) greater than or equal to $\frac{E(S_n^2)}{c^2}$

(B) less than or equal to $E(S_n^2)/c^2$

(C) less than or equal to $c^2 E(S_n^2)$

(D) less than or equal to $E(S_n^2)/c$

46. Let X and Y have joint mass function $f(x, y) = (x(x+1))^{-1}$, for $x = 1, 2, ...$ and $y = 1, 2, ...$ Then identify the correct one from below :

(A) $E(Y|X) = \infty, E(Y) = \infty$

(B) $E(Y|X) = \infty$, but $E(Y) < \infty$

(C) $E(Y|X) < \infty$, but $E(Y) = \infty$

(D) None of the above

47. Let N be Poisson distributed with parameter λ. If $S = \sum_{r=1}^{N} X_r$, where $\{X_r : r \ge 0\}$ are independent identically distributed non-negative integer-valued random variables then choose the correct answer of $E[Sg(s)]$ from the following :

(A) $E(g(s+X_0)X_0)$

(B) $\lambda E(g(s)X_0)$

(C) $\lambda E(g(s+X_0)X_0)$

(D) None of the above

48. Let X and Y have joint density function

$f(x, y) = \frac{1}{4}\{1 + xy(x^2 - y^2)\}$,

$|x| < 1, |y| < 1$.

Then the characteristic function of $X+Y$ satisfies the following: (Choose the correct answer)

(A) $\phi_{X+Y}(t) = \phi_X(t)\phi_Y(t)$ and X and Y are independent

(B) $\phi_{X+Y}(t) \ne \phi_X(t)\phi_Y(t)$

(C) $\phi_{X+Y}(t) = \phi_X(t)\phi_Y(t)$ and X and Y are dependent

(D) None of the above

49. Let X and Y be independent $N(0, 1)$ variables, and let U and V be independent of X and Y. Choose the correct distribution of the variable $Z = (UX + VY)/\sqrt{U^2 + V^2}$ from the following :

(A) Z has χ^2 distribution

(B) Z has the $N(0, 1)$ distribution

(C) Z has beta distribution of 1st kind

(D) None of the above

50. Let X have the $\Gamma(\lambda, m)$ distribution and let Y be independent of X with the beta distribution with parameters n and $m-n$, where m and n are non-negative integers satisfying $n \leq m$. Choose the correct distribution of $Z = XY$ from the following :

(A) Beta distribution of 1st kind

(B) Normal distribution

(C) $\Gamma(\lambda, n)$ distribution

(D) Beta distribution of 2nd kind

51. Let F be a distribution function of a continuous random variable X and r be a positive integer. Then, identify the wrong statement from the below :

(A) $f(x) = F'(x)$ is the pdf of X, if the derivative exists

(B) $(F(x))^r$ is a distribution function

(C) $1-(1-F(x))^r$ is a distribution function

(D) The derivative $F'(x)$ should always exist and $0 \leq F'(x) \leq 1$

52. Let F and G are distribution functions and $0 \leq \lambda \leq 1$. Then choose the correct alternative for H where $H = \lambda F + (1-\lambda) G$:

(A) H is non-decreasing, but not right continuous

(B) H is non-increasing but right continuous

(C) H is non-decreasing as well as right continuous

(D) H is a distribution function only if $\lambda = \frac{1}{2}$

53. Let $\phi(t)$ be the characteristic function of a random variable X, also let $E(X)$ exist. Then identify the correct alternative from the following :

(A) $|\phi(t)| \leq E(|tX|)$

(B) $|1-\phi(t)| \geq E(|tX|)$

(C) $|1-\phi(t)| \leq E(|tX|)$

(D) $|\phi(t)| \leq E(X^t)$

54. Let $X_1, X_2, \ldots, X_n$ be jointly normal with $E(X_i)=0$ and $E(X_i^2)=1$ for all i and $cov(X_i, X_j)=\rho$ if $|j-i|=1$ and $=0$, otherwise. Let $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{S_n}{n}$. Then WLLN holds for

(A) $\{X_n\}$, *i.e.*, $\frac{S_n}{n} \xrightarrow{p} 0$ as $n \to \infty$

(B) $\{X_n\}$, *i.e.*, $\frac{S_n}{n} \xrightarrow{p} 1$, as $n \to \infty$

(C) $\{X_n\}$, *i.e.*, $\frac{S_n}{n} \xrightarrow{p} 1$ for all n

(D) None of the above

55. In Bayesian approach to testing $H_0 : \theta = \theta'$ against $H_1 : \theta = \theta''$, let $L(\theta)$ be the likelihood and $h(\theta)$ be the prior probability density function of θ. Also, suppose $\mathscr{L}(\theta, \delta)$ represents the loss function associated with the decision δ, when θ is the true value.
The Bayes' solution requires that the decision $\delta = \theta''$ be taken if (identify the correct one)

(A) $\frac{L(\theta')}{L(\theta'')} < \frac{\mathscr{L}(\theta'', \theta')\, h(\theta'')}{\mathscr{L}(\theta', \theta'')\, h(\theta')}$

(B) $\frac{L(\theta')}{L(\theta'')} > \frac{\mathscr{L}(\theta'', \theta')\, h(\theta'')}{\mathscr{L}(\theta', \theta'')\, h(\theta')}$

(C) $\frac{L(\theta')}{L(\theta'')} < \frac{\mathscr{L}(\theta'', \theta')\, h(\theta')}{\mathscr{L}(\theta', \theta'')\, h(\theta'')}$

(D) None of the above

56. Let $X_1, X_2, \ldots, X_n$ $(n=100)$ denote a random sample of size 100 from $N(\theta, 100)$. We want to test $H_0 : \theta = 75$ against $H_1 : \theta = 78$.
Let $\mathscr{L}(\theta, \delta)$ be the loss function associated with the decision δ, when θ is the true value. For $\mathscr{L}(75, 78) = 3$ and $\mathscr{L}(78, 75) = 1$, a minimax solution is given by (identify the correct one)

(A) $\bar{X} \geq 70{\cdot}0$

(B) $\bar{X} \leq 71{\cdot}8$

(C) $\bar{X} \geq 76{\cdot}8$

(D) $\bar{X} \geq 0$

57. Consider the following statements :

(a) If T is the maximum likelihood (ML) estimator of the parameter θ and $\psi(\theta)$ is a one-to-one function of θ, then $\psi(T)$ is not the ML estimator for $\psi(\theta)$.

(b) If T be a sufficient statistic for θ, then any solution of the likelihood equation will be a function of t.

(c) A consistent solution of the likelihood equation is not unique.

(d) ML estimators are always unbiased.

Use the truth table given below to choose the correct answer.

Codes :	(a)	(b)	(c)	(d)
(A)	True	False	False	False
(B)	False	False	True	False
(C)	False	True	False	False
(D)	False	False	False	True

58. Match the following four methods with their correct uses and select the correct match from the codes below :

Method	Uses
(a) All problems of statistical inference may be looked upon as problems of	(i) should be taken when $\underset{\sim}{x}$ is the set of observations for varying $\underset{\sim}{x}$
(b) Any decision procedure would dictate which decision	(ii) decision making in the face of uncertainty
(c) According minimax approach one tries to guard against the worst	(iii) is a convex loss function
(d) Usually most plausible type of loss function	(iv) by requiring that the chosen decision rule should provide maximum protection against the highest possible risk

Codes :	(a)	(b)	(c)	(d)
(A)	(ii)	(i)	(iv)	(iii)
(B)	(i)	(ii)	(iii)	(iv)
(C)	(ii)	(iv)	(iii)	(i)
(D)	(iii)	(ii)	(i)	(iv)

59. Let W_n denote a random variable with mean μ and variance b/n^p, where $p > 0$, μ and b all are constants independent of n. Then identify the correct alternative :

(A) W_n does not converge to μ in probability

(B) W_n does converge to μ in probability

(C) W_n is degenerate, with mean zero

(D) None of the above

60. Let the random variable Z_n have a Poisson distribution with parameter $\theta = n$. Then, which of the following statistics will have a limiting $N(0, 1)$ distribution ?

(A) $(Z_n - n)/n$

(B) $(Z_n - 2n)/n^2$

(C) $(Z_n - n)/\sqrt{n}$

(D) $(Z_n - \sqrt{n})/\sqrt{n}$

61. Let $\bar{X}_n$ and S_n^2 denote, respectively, the mean and variance of a random sample of size n from $N(\mu, \sigma^2)$, $\sigma^2 > 0$.

Then, $\sigma_{\bar{X}_n}/S_n$ converges in probability to (identify the correct alternative) :

(A) zero

(B) unity

(C) μ

(D) σ

62. Let Y_n denote a random variable that is $b(n, p)$, $0 < p < 1$. Then, the statistic

$$\left(\frac{Y_n}{n}\right)\left(1 - \frac{Y_n}{n}\right) \Big/ (p(1-p))$$

converges in probability to (identify the correct one) :

(A) zero

(B) p

(C) $p(1-p)$

(D) unity

63. Let (X_n, Y_n) follow a bivariate normal distribution $N(\mu_1, \mu_2; \sigma_1^2, \sigma_2^2; \rho)$ with $\rho = 1 - \frac{1}{n}$.

Then the limiting distribution of the conditional distribution of Y_n, given $X_n = x$, is given by (identify the correct one) :

(A) $N(x, \sigma_2^2)$

(B) $N\left(\mu_2 + \left(\frac{\sigma_2}{\sigma_1}\right)(x - \mu_1), \sigma_2^2\right)$

(C) Degenerate at $\mu_2 + \left(\frac{\sigma_2}{\sigma_1}\right)(x - \mu_1)$

(D) None of the above

64. If $A \sim W_p(\Sigma, n)$ and B is a $q \times p$ matrix, then choose the correct answer of BAB' from the following :

(A) $BAB' \sim W_p(B\Sigma B', n)$

(B) $BAB' \sim W_p(\Sigma, n)$

(C) $BAB' \sim W_p(I, n)$

(D) None of the above

65. If $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$ and if rank $(\Sigma) = k (< p)$, Σ^{-1} does not exist. Consider that $\lambda_1, \lambda_2, \ldots, \lambda_k$ are the k eigenvalues of Σ. Then the distribution of X is

(A) $$\frac{1}{(2\pi)^{k/2}(\lambda_1 \lambda_2 \ldots \lambda_k)^{1/2}} \exp\left[-\frac{1}{2}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \Sigma^{-1}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)\right]$$

(B) $$\frac{1}{(2\pi)^{p/2}(\lambda_1 \lambda_2 \ldots \lambda_p)^{1/2}} \exp\left[-\frac{1}{2}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \Sigma^{-1}\left(\underset{\sim}{X} - \mu\right)\right]$$

(C) $$\frac{1}{(2\pi)^{p/2}(\lambda_1 \lambda_2 \ldots \lambda_k)^{1/2}} \exp\left[-\frac{1}{2}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \Sigma^{-1}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)\right]$$

(D) None of the above

66. If X_1 and X_2 are independent data matrices of order $(n_1 \times p)$ and $(n_2 \times p)$, respectively and if the n_i rows of $X_i (i = 1, 2)$ are identically and independently distributed as $N_p(\mu_i, \Sigma_i)$, then for $\underset{\sim}{\mu}_1 = \underset{\sim}{\mu}_2$ and $\Sigma_1 = \Sigma_2$.

(A) $\frac{n_1 n_2}{n_1 + n_2} D^2 \sim F(p, n-2)$ variable

(B) $\frac{n_1 n_2}{n_1 + n_2} D^2 \sim T^2(p, n-2)$ variable

(C) $\frac{n_1 n_2}{n_1 + n_2} D^2 \sim T^2(p-1, n-3)$ variable

(D) None of the above

where

$D^2 = (\overline{X}_1 - \overline{X}_2)' S_u^{-1} (\overline{X}_1 - \overline{X}_2)$

called sample Mahalanobis distance and

$S_u = (n_1 S_1 + n_2 S_2)/(n_1 + n_2 - 2)$.

67. Let $X'BX$ and $X'CX$ be two quadratic forms of $X_1, X_2, ..., X_p$, where

$X = (X_1\ X_2 \ldots X_p)'$ and the pdf of X is

$$f\left(\underset{\sim}{x}\right) = \frac{|A|^{\frac{1}{2}}}{(2\pi)^{p/2}} exp\left\{-\frac{1}{2} X'AX\right\}.$$

Choose the necessary condition for the independence of the two quadratic forms from the following

(A) $BA = 0$

(B) $BC = 0$

(C) $BA^{-1}C = 0$

(D) $CA^{-1} = 0$

68. The regression equation of Y on X may be defined as (identify the correct one) :

(A) $E(X|Y)$

(B) $E(Y|X)$

(C) $E(Y + \alpha X)$, (α = constant)

(D) $E(X + \beta Y)$, (β = constant)

69. Consider the Gauss-Markov linear model given by $E(\underline{Y}) = \underline{A}\underline{\beta}$, $D(\underline{e}) = \sigma^2 I$, $(\sigma^2 > 0)$. Then , a necessary and sufficient condition for a parametric function $\underline{e}'\underline{\beta}$ to be estimable is that (identify the correct one) :

(A) $Rank\ (A') = Rank(A'\underline{e})$

(B) $Rank\ (A') > Rank(A'\underline{e})$

(C) $Rank\ (A') < Rank(A'\underline{e})$

(D) $Rank\ (A') = Rank(I)$

I being the identity matrix of an appropriate order.

70. Consider the Gauss-Markov Model given by $E(\underline{Y}) = A\underline{\beta}$, $D(\underline{e}) = \sigma^2 I$, where the symbols have their usual meanings.

If A is of full rank, then the BLUE of $\underline{\beta}$ is given by (identify the correct one) :

(A) $(AA')\,A\underline{Y}$

(B) $A'A\underline{Y}$

(C) $(AA')^{-1}A\underline{Y}$

(D) $(A'A)^{-1}A'\underline{Y}$

71. Let Y_1, Y_2, Y_3, Y_4 are independent with

$$E(Y_1) = E(Y_2) = \theta_1 + \theta_2$$
$$E(Y_3) = E(Y_4) = \theta_1 + \theta_3$$
$$V(Y_i) = \sigma^2,\ i = 1, 2, 3, 4$$

Then the condition of estimability of the parametric function $\underline{l}'\underline{\theta} = l_1\theta_1 + l_2\theta_2 + l_3\theta_3$ is (identify the correct one) :

(A) $l_1 = l_2 = l_3$

(B) $l_1 + l_2 = l_3$

(C) $l_1 = l_2 + l_3$

(D) $2l_1 = l_2 + l_3$

72. If the coefficient of variations of X and Y be the same, then in large samples, the ratio estimator has a smaller variance compared to mean per unit if (identify the correct answer) :

(A) $\rho > 0{\cdot}5$

(B) $\rho > 0{\cdot}25$

(C) $-0{\cdot}25 \le \rho \le 0{\cdot}25$

(D) $0 \le \rho \le 0{\cdot}5$

ρ being the correlation coefficient between X and Y.

73. Identify the error, which is not a non-sampling error, from the following :

(A) Errors arising from faulty sampling frame

(B) Errors arising from faulty recording of data

(C) Errors arising from fatigue of the enumerator

(D) Errors arising from studying a part of the population

74. The regression estimator is more precise than the ratio estimator except when (identify the correct answer) :

(A) The relation between X and Y is quadratic

(B) The relation between X and Y is a straight line through the origin

(C) The correlation coefficient ρ is + 1, or –1

(D) None of the above

75. In applying double sampling for regression, let the cost function be given by $C_0 = cn + c'n'$, where n' and n be the size of the first and second sample respectively.

Then the critical ranges of c/c' that make double sampling profitable is (identify the correct answer) :

(A) $\frac{c}{c'} > \frac{\left(1+\sqrt{1-\rho^2}\right)^2}{\rho^2}$

(B) $\frac{c}{c'} < \frac{\left(1+\sqrt{1-\rho^2}\right)^2}{\rho^2}$

(C) $\frac{c}{c'} > \frac{\left(1-\sqrt{1-\rho^2}\right)^2}{\rho^2}$

(D) $\frac{c}{c'} > \frac{\left(1-\rho^2\right)}{\left(1+\rho^2\right)}$

where ρ is the correlation coefficient between X and Y, X being the auxiliary variable.

76. Consider the following statements :

(a) The term factorial experiment is coined by Karl Pearson.

(b) In a 2^3 factorial experiment there are 3 first order interactions.

(c) In a 2^2 factorial experiment the factorial effect total of $[A]=[a]+[b]-[ab]-[1]$.

(d) In a 2^3 factorial experiment having r replications the error df is $7r-1$.

Use the true table below to choose the correct answer.

Codes :

	(a)	(b)	(c)	(d)
(A)	True	True	False	False
(B)	False	True	False	True
(C)	True	False	False	False
(D)	False	True	False	False

77. Standard error of a factorial effect total is (choose the correct answer from the following)

(A) $\sqrt{4r\sigma_e^2}$

(B) $\sqrt{r\sigma_e^2}$

(C) $\sqrt{4\sigma_e^2}$

(D) None of the above

where r and σ_e^2 represent the replication and common variance respectively.

78. Partial confounding essentially means that

(A) different effects are confounded in different replications

(B) same set of effects are confounded in all replications

(C) the same effect is confounded in at least two replications

(D) All of the above

79. For a BIBD with parameters v, b, r, k, λ, which of the designs given below is not a BIBD ?

(A) 6, 10, 5, 3, 2

(B) 7, 7, 3, 3, 1

(C) 8, 14, 7, 4, 2

(D) 7, 7, 4, 4, 2

80. Match the following four methods with their correct uses and select the correct match from the codes below :

Method	Uses
(a) In an incomplete block design we	(i) generalised case of balanced incomplete block design (BIBD)
(b) Partially balanced incomplete block design is a	(ii) five parameters
(c) in a balanced incomplete block design we have	(iii) can be used to construct the balanced incomplete block design
(d) A number of mutually orthogonal latin squares (MOLS)	(iv) give emphasis to pair of treatments also

Codes :

	(a)	(b)	(c)	(d)
(A)	(i)	(ii)	(iii)	(iv)
(B)	(ii)	(i)	(iv)	(iii)
(C)	(iv)	(i)	(ii)	(iii)
(D)	(iii)	(ii)	(i)	(iv)

81. Considering Assertion (Ass) and Reason (R) choose the correct answer from the given codes :

Assertion (Ass) : In a 2^5 factorial experiment confounding is necessary.

Reason (R) : Because if we allot all the 32 treatment combinations in a single block then block size may not remain homogeneous and error will creep into the design.

Codes :

(A) (Ass) is false, but (R) is true

(B) (Ass) is true, but (R) is false

(C) Both (Ass) and (R) are true

(D) Both (Ass) and (R) are false

82. Let $\{Z_n\}$ be a sequence of uncorrelated real-valued variables with means zero and variances unity. Define $Y_n = \alpha Y_{n-1} + Z_n$, $-\infty < n < \infty$, $|\alpha| < 1$. Then Y has the autocovariance function of $\log m$ given by (identify the correct one) :

(A) $\alpha^m / (1-\alpha^2)$

(B) $\alpha^{2m} / (1-\alpha)^2$

(C) $\alpha^{|m|} / (1-\alpha^2)$

(D) $\alpha^{|m|} / (1-\alpha)^2$

83. Let $\{Z_n : n = ..., -1, 0, 1, ...\}$ be independent random variables with zero means and unit variances, and define the moving average process

$$X_n = \frac{Z_n + aZ_{n-1}}{\sqrt{1+a^2}}, \quad |a| < 1.$$

Then the autocovariance function of $\log k$, with $k = -1$, or 1 (identify the correct one) is

(A) 0

(B) 1

(C) $a/(1+a^2)$

(D) $(1+a)/(1+a^2)$

84. Let $..., Z_{-1}, Z_0, Z_1, Z_2, ...$ be independent real random variables with means 0 and variances 1, and let $\alpha, \beta \in \mathbb{R}$. Then the sequence $\{W_n\}$ is weakly stationary, where $W_n = \alpha W_{n-1} + \beta W_{n-2} + Z_n$, $n = ..., -1, 0, 1, ...$, if the zeros of the equation $Z^2 - \alpha Z - \beta = 0$ are (identify the correct answer)

(A) greater than 1 in absolute value

(B) between -1 and 1

(C) all equal and greater than 1

(D) all distinct and less than -1

85. A Markov chain with state space 0, 1, 2, 3 is given by the following transition probability matrix :

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Then identify the incorrect assertion from the following :

(A) The Markov chain is not irreducible

(B) The communicating classes are {0, 1, 2} and {3}

(C) The state 3 is an absorbing state

(D) The matrix is a doubly stochastic matrix

86. Consider the Markov Chain (M.C.) with the transition probability matrix given by

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

Then identify the wrong assertion from the following :

(A) The M.C. is irreducible

(B) The states are periodic with period 2

(C) The states are persistent non-null

(D) The states are persistent null

87. Let $N(t)$ be a Poisson process with mean λt. Then go through the following statements and identify the wrong one :

(A) The interarrival times between successive occurrences are independent

(B) The interarrival time follows the negative exponential law with mean $\frac{1}{\lambda}$

(C) The interarrival time follows the negative exponential law with mean λ.

(D) The difference of two independent Poisson processes is not a Poisson process

88. Two statements are given below — One is Assertion (Ass) and the other is the Reason (R).

Assertion (Ass) : In population projection even if the assumption of linear growth is less realistic than that of exponential growth, yet one may prefer the formar.

Reason (R) : Linear models are simpler.

Select your answer from the following codes :

(A) (Ass) and (R) both are true

(B) (Ass) is true but (R) is not

(C) Both (Ass) and (R) are false

(D) (Ass) is false but (R) is true

89. In a life table, using the standard notation for various quantities used, one can express T_x, the total number of years lived by the cohort after attaining the age x, as the following :
(Choose the correct one)

(A) $L_x + T_{x+1}$

(B) $\frac{d_x}{l_x}$

(C) $L_x + L_{x+1}$

(D) $\frac{l_{x+1}}{l_x}$

90. Choose the correct age-specific fertility rate ${}_ni_x$, for the age group x to $x+n-1$ from the following :

(A) ${}_ni_x = \frac{B_x}{{}_nf_x} \times 1000$

(B) ${}_ni_x = \frac{{}_nB_x}{{}_nf_x} \times 1000$

(C) ${}_ni_x = \frac{{}_nB_x}{{}_nf_x} \times 1000$

(D) ${}_ni_x = \frac{{}_nf_x}{{}_nf_x} \times 1000$

where B_x is the number of life births to women of age x, ${}_nB_x$ represents the number of life births to women of age x to $x+n-1$ and ${}_nf_x$ represents the number of life births to women of age x to $x+n-1$ in the given region during the given period, and ${}_nf_x$ represents the number of women of age x to $x+n-1$ in the region during the given period.

91. Considering Assertion (Ass) and Reason (R) choose the correct answer from the given codes.

Assertion (Ass) : Net reproduction rate is always less than equal to gross reproduction rate.

Reason (R) : Since Net reproduction rate considers both mortality and migration into account.

Codes :

(A) (Ass) is false, but (R) is true

(B) (Ass) is true, but (R) is false

(C) Both (Ass) and (R) are true

(D) Both (Ass) and (R) are false

92. Consider the following statements :

(a) Greville's method is used to construct abridged life tables.

(b) In the calculation of gross reproduction rate we use the life tables.

(c) Age specific death rates are not used in the construction of life tables.

(d) Life tables are not used in the population projection.

Use the truth table given below to choose the correct answer.

Codes :	(a)	(b)	(c)	(d)
(A)	True	False	False	False
(B)	False	True	False	True
(C)	True	True	False	False
(D)	False	False	False	True

93. Consider the exponential lifetime distribution given by

$$f(t) = \lambda e^{-\lambda t},\ t > 0,\ \lambda > 0.$$

Then identify the correct alternative from the following :

(A) The failure rate is a quadratic function of t

(B) The failure rate is IFR type

(C) The failure rate is DFR type

(D) The failure rate is both IFR and DFR type

94. Let us assume that the state of the component $i\ (i = 1, 2, \ldots, n)$ is described by a binary random variable X_i, defined by

$P(X_i = 1) = p_i,\ P(X_i = 0) = 1 - p_i,$

where '1' and '0' correspond to the operational and failure state, respectively.

Consider now a parallel system which works if at least one component works. The reliability of such a system is given by (choose the correct alternative) :

(A) $1 - \prod_{i=1}^{n} p_i$

(B) $e^{-\sum_{i=1}^{n} p_i}$

(C) $1 - \prod_{i=1}^{n} (1 - p_i)$

(D) $1 - \sum_{i=1}^{n} p_i$

95. In order to judge the relative merits of two or more rival sequential tests, we make use of two criteria. These are :

(A) Operating Characteristic (OC) function and the Average Sample Number (ASN) function

(B) OC function and any probability function

(C) ASN function and any probability function

(D) None of these

96. Two statements are given below, one is Assertion (Ass) and the other is Reason (R).

Assertion (Ass) : p-control chart for fraction defective is obtained for same population proportion P, when P' is known value of P, as

U.C. $L_p = P' + A\sqrt{P'(1-P')}$

L.C. $L_p = P' - A\sqrt{P'(1-P')}$

C. $L_p = P'$,

where $A = 3/\sqrt{n}$.

Reason (R) : p-control chart for fraction defective is obtained for attributes.

Choose the correct answer from the following codes :

Codes :

(A) Both (Ass) and (R) are true

(B) (Ass) is true but (R) is false

(C) Both (Ass) and (R) are false

(D) (Ass) is false but (R) is true

97. Consider $M(\lambda)/G/1$ in equilibrium.

The mean waiting time of an arriving customer is (identify the correct one) :

(A) $\frac{1}{2}\lambda/(1-\rho)$

(B) $\frac{1}{2}\lambda E(s^2)/(1-\rho)$

(C) $\frac{1}{2}\lambda E(s^2)(1-\rho)$

(D) $\frac{1}{2}\lambda(1-\rho)/E(s^2)$

where s is a typical service time and $\rho = \lambda E(s)$.

98. In a M/D/2 model (with usual assumptions) it is given that the service time of each customer is 20 minutes. Both the servers work with equal efficiency. Then in two hours, the number of customers served by the service facility will be

(A) 12 customers

(B) 20 customers

(C) 6 customers

(D) 3 customers

99. In a machine maintenance model (with usual assumptions), assume that the number of mechanics (or servers) is 3 and that there are 10 machines. Also suppose that at a certain point of time, the number of customers in the system is 2. Then the arrival rate into the system would be

(A) 8λ

(B) 10λ

(C) 7λ

(D) Cannot be determined

where λ is the rate at which a machine may break down.

100. Let

$X_n = A\cos(n\lambda) + B\sin(n\lambda)$,

where A and B are uncorrelated random variables with zero means and unit variances. Then identify the correct answer from the following :

(A) $\{X_n\}$ is stationary with variance 1

(B) $\{X_n\}$ is stationary with variance λ

(C) $\{X_n\}$ is non-stationary

(D) None of the above