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Test Booklet Series **A**

T. B. C. : OTE – 8/18

TEST BOOKLET

MATHEMATICS

PAPER – II

Sl No. 2701

Time Allowed : 3 Hours

Maximum Marks : 200

: INSTRUCTIONS TO CANDIDATES :

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET OF THE SAME SERIES ISSUED TO YOU.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C** OR **D**, AS THE CASE MAY BE, IN THE APPROPRIATE PLACE IN THE ANSWER SHEET USING BALL POINT PEN (BLUE OR BLACK).
3. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write *anything else* on the Test Booklet.
4. **YOU ARE REQUIRED TO FILL UP & DARKEN ROLL NO., TEST BOOKLET / QUESTION BOOKLET SERIES IN THE ANSWER SHEET AS WELL AS FILL UP TEST BOOKLET / QUESTION BOOKLET SERIES AND SERIAL NO. AND ANSWER SHEET SERIAL NO. IN THE ATTENDANCE SHEET CAREFULLY. WRONGLY FILLED UP ANSWER SHEETS ARE LIABLE FOR REJECTION AT THE RISK OF THE CANDIDATE.**
5. This Test Booklet contains **100** items (questions). Each item (question) comprises four responses (answers). You have to select the correct response (answer) which you want to mark (darken) on the Answer Sheet. In case, you feel that there is more than one correct response (answer), you should mark (darken) the response (answer) which you consider the best. In any case, choose **ONLY ONE** response (answer) for each item (question).
6. You have to mark (darken) all your responses (answers) **ONLY** on the **separate Answer Sheet** provided by **using BALL POINT PEN (BLUE OR BLACK)**. See instructions in the Answer Sheet.
7. All items (questions) carry equal marks. All items (questions) are compulsory. Your total marks will depend only on the number of correct responses (answers) marked by you in the Answer Sheet.
8. Before you proceed to mark (darken) in the Answer Sheet the responses to various items (questions) in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per the instructions sent to you with your **Admission Certificate**.
9. After you have completed filling in all your responses (answers) on the Answer Sheet and after conclusion of the examination, you should hand over to the Invigilator the *Answer Sheet* issued to you. You are allowed to take with you the candidate's copy / second page of the Answer Sheet along with the **Test Booklet**, after completion of the examination, for your reference.
10. Sheets for rough work are appended in the Test Booklet at the end.

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SEAL

1. Let $A = \begin{pmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x^2-1) \end{pmatrix}$.

The f(x) = Det (A), then f(200) is equal to :

(A) 1

(B) 2

(C) 3

(D) None of these

2. Let $A = \begin{pmatrix} \log_a\left(\frac{x}{y}\right) & \log_a\left(\frac{y}{z}\right) & \log_a\left(\frac{z}{x}\right) \\ \log_b\left(\frac{y}{z}\right) & \log_b\left(\frac{z}{x}\right) & \log_b\left(\frac{x}{y}\right) \\ \log_c\left(\frac{z}{x}\right) & \log_c\left(\frac{x}{y}\right) & \log_c\left(\frac{y}{z}\right) \end{pmatrix}$.

The value of the Det(A) is :

(A) 1

(B) –1

(C) None of these

(D) $\log_a(xyz)$

3. If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$ and $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is equal to $\lambda(\vec{b} \times \vec{c})$, then λ is equal to :

(A) 3

(B) 4

(C) 5

(D) 6

4. Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}$, $\vec{b} = 2\hat{i} - 6\hat{j}$ and $\vec{c} = 2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$. Then the resolution of the vector $\vec{\alpha}$ into components with respect to $\vec{a}$ and $\vec{b}$ is given by :

(A) $\vec{\alpha} = 2\vec{a} - 3\vec{b}$

(B) $\vec{\alpha} = 3\vec{a} - 2\vec{b}$

(C) $\vec{\alpha} = 2\vec{a} + 3\vec{b}$

(D) $\vec{\alpha} = 3\vec{a} + 2\vec{b}$

5. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is :

(A) 1

(B) 2

(C) 3

(D) 4

6. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is :

(A) tan u

(B) – tan u

(C) $-\frac{1}{2}\tan u$

(D) $\frac{1}{2}\tan u$

7. If $\int f(x) \cos x dx = \frac{1}{2}\{f(x)\}^2 + c$, then f(x) is :

(A) $\sin x + c$

(B) $x + c$

(C) $\cos x + c$

(D) None of these

8. If $\int_0^x f(t)dt = x + \int_x^1 t\, f(t)dt$, then the velue of f(1) is :

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{5}$

9. Let $f(x) = \min(|x|, 1 - |x|, \frac{1}{4})$ for every real x, then the value of $\int_{-1}^{1} f(x) dx$ is equal to :

(A) $\frac{1}{8}$

(B) $\frac{3}{8}$

(C) $\frac{5}{8}$

(D) $\frac{7}{8}$

10. Given that n is odd and m is an even integer. The value of $\int_0^\pi \cos mx \sin nx dx$ is :

(A) $\frac{2n}{n^2 - m^2}$

(B) $\frac{2m}{n^2 - m^2}$

(C) $\frac{m}{n^2 - m^2}$

(D) $\frac{n}{n^2 - m^2}$

11. Suppose for every integer n $\int_n^{n+1} f(x)dx = n^2$, the value of $\int_{-2}^{4} f(x) dx$ is :

(A) 15

(B) 17

(C) 19

(D) None of these

12. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^5 = \ln\left(\frac{d^2y}{dx^2}\right)$ respectively are :

(A) Order 2 and degree is 5

(B) Order is not defined but degree is 2

(C) Order is 2 but degree is not defined

(D) Order 5 and degree 2

13. The solution of the differential equation $\frac{dy}{dx} = \frac{y(x - y\ln y)}{x(x\ln x - y)}$ is :

(A) $\frac{x\ln x + y\ln y}{xy} = c$

(B) $\frac{x\ln x - y\ln y}{xy} = c$

(C) $\frac{\ln x}{x} + \frac{\ln y}{y} = c$

(D) $\frac{\ln x}{x} - \frac{\ln y}{y} = c$

(where c is any arbitrary constant)

14. The solution of the differential equation $dy + y\tan x dx = \sin 2x dx$, $y(0) = 1$ is :

(A) $y = 3\cos x - 2\cos^2 x$

(B) $y = 2\sin x - 3\cos^2 x$

(C) $y = 2\cos x + 3\sin^2 x$

(D) $y = 3\sin x - 2\sin^2 x$

15. If $y = f(x)$ passing through (1, 2) satisfies the differential equation $y(1 + xy)dx - xdy = 0$, then :

(A) $f(x) = \frac{2x}{2 - x^2}$

(B) $f(x) = \frac{4x}{1 - 2x^2}$

(C) $f(x) = \frac{x + 1}{x^2 + 1}$

(D) $f(x) = \frac{x - 1}{4 - x^2}$

16. The inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$ is :

(A) $\frac{e^t - e^{-t}}{t}$

(B) $\frac{e^t + e^{-t}}{t}$

(C) $\frac{e^t + e^{-t}}{t^2}$

(D) $\frac{e^t - e^{-t}}{t^2}$

17. The Laplace transform of $f(t) = \frac{\cos\sqrt{t}}{\sqrt{t}}$ is given by :

(A) $\sqrt{\frac{\pi}{s}}\, e^{-\frac{1}{3s}}$

(B) $\sqrt{\frac{\pi}{s}}\, e^{-\frac{1}{2s}}$

(C) $\sqrt{\frac{\pi}{s}}\, e^{-\frac{1}{4s}}$

(D) $\sqrt{\frac{\pi}{s}}\, e^{-\frac{1}{s}}$

18. If $f(x, y) = \ln\left(\frac{x^4 + y^4}{x + y}\right)$, then the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is :

(A) 1

(B) 2

(C) 3

(D) 4

19. If $z = e^{xy^2}$, $x = t\cos t$, $y = t\tan t$, then the value of $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$ is :

(A) $\frac{\pi^3}{8}$

(B) $-\frac{\pi^2}{8}$

(C) $\frac{\pi^2}{8}$

(D) $-\frac{\pi^3}{8}$

20. The value of the integral $\int_0^2 \int_0^{\frac{y^2}{2}} \frac{y}{\sqrt{x^2 + y^2 + 1}}\, dx\, dy$ is equal to :

(A) $\frac{5}{4}\ln 5 + 1$

(B) $\frac{5}{4}\ln 5 - 2$

(C) $\frac{5}{4}\ln 5 - 1$

(D) $\frac{5}{4}\ln 5 + 2$

21. The value of the integral $\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx$ is :

(A) $\frac{2n}{4n^2 - 1}$

(B) $\frac{n}{4n^2 - 1}$

(C) $\frac{n}{n^2 - 1}$

(D) $\frac{2n}{n^2 - 1}$

($P_n(x)$ is the Legendre poly-nomial of degree n)

22. The values of a, b and c, so that $f = (x - 2y + az)\hat{i} + (bx - y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational are respectively :

(A) 2, –1, 4

(B) 4, 2, –1

(C) –1, 4, 2

(D) 2, 4, –1

23. The solution of the differential equation $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$ is :

(A) $y = x \tan \log(cx)$

(B) $y = x \tan \log x + c$

(C) $y = x + \tan \log x + c$

(D) $y = x - \tan \log (cx)$

24. The solution of the differential equation $y = x\frac{dy}{dx} + \sin^{-1}\left(\frac{dy}{dx}\right)$ is :

(A) $y = cx - \sin^{-1} c$

(B) $y = x + \sin^{-1} c$

(C) $y = cx + \sin^{-1} c$

(D) $y = x^2 + \sin c$

25. The general solution of the differential equation $\frac{d^2y}{dx^2} + y = \sec x$ is given by :

(A) $[c_1 + \ln|\cos x|] \cos x + (c_2 + x) \sin x$

(B) $[c_1 - \ln|\cos x|] \cos x + (c_2 + x) \sin x$

(C) $[c_1 - \ln|\cos x|] \cos x - (c_2 + x) \sin x$

(D) $[c_1 + \ln|\cos x|] \cos x + (c_2 - x) \sin x$

26. Consider the following differential equations :

(i) $$x^2\left(\frac{d^2y}{dx^2}\right)^6 + y^{-\frac{2}{3}}\left\{1+\left(\frac{d^3y}{dx^3}\right)^5\right\}^{\frac{1}{2}} + \frac{d^2}{dx^2}\left\{\left(\frac{d^2y}{dx^2}\right)^{-\frac{2}{3}}\right\} = 0$$

(ii) $$\frac{dy}{dx} - 6x = \left\{ay + bx\left(\frac{dy}{dx}\right)\right\}^{-\frac{2}{3}}, b \neq 0.$$

The sum of the order of the first differential equation and degree of the second differential equation is :

(A) 6

(B) 7

(C) 8

(D) 9

27. If $y_1 = x^m$ and $y_2 = x^n$ where m and n are constants, are two solutions of a second order differential equation with constant coefficients, then $y = c_1y_1 + c_2y_2$ is the general solution of the same equation if :

(A) $m = n$

(B) $m \neq n$

(C) $m + n = 1$ only

(D) $m = -n$ only

28. The solution of the differential equation $\frac{dy}{dx} = \sec(x + y)$ is :

(A) $y - \tan\frac{1}{2}(x + y) = C$

(B) $y + \tan\frac{1}{2}(x + y) = C$ ***

(C) $y - \tan\frac{1}{2}(x - y) = C$

(D) $y + \tan\frac{1}{2}(x - y) = C$ ***

where C is any arbitrary constant.

29. The solution of the differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$, is :

(A) $y = Cxe^{\frac{1}{xy}}$

(B) $y = Cxe^{\frac{1}{x}}$

(C) $x = Cye^{\frac{1}{y}}$

(D) $x = Cye^{\frac{1}{xy}}$

where C is any arbitrary constant.

30. The solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$, $y(0) = 2$, $y'(0) = 0$ is :

(A) $y = 2\sin 2x$

(B) $y = 2\cos 2x$

(C) $y = \cos 2x + \sin 2x$

(D) $y = \cos 2x - \sin 2x$

31. The Laplace transform of $f(t) = t^{\frac{7}{2}}e^{3t}$ is :

(A) $\frac{103\sqrt{\pi}}{16(s-3)^{9/2}}$

(B) $\frac{105\sqrt{\pi}}{15(s-3)^{9/2}}$

(C) $\frac{105\sqrt{\pi}}{16(s-3)^{7/2}}$

(D) $\frac{105\sqrt{\pi}}{16(s-3)^{9/2}}$

32. If A is real symmetric and $\lambda \neq \mu$ are two eigen values of A with corresponding eigen vector x and y respectively then :

(A) $x \| y$

(B) $x \perp y$

(C) $x = y^T$

(D) None of these

33. Every square matrix can be expressed as the sum of a :

(A) Hermitian and Skew-Hermitian matrix

(B) Symmetric and Skew-Symmetric matrix

(C) Hermitian and Symmetric matrix

(D) Skew-Hermitian and Skew-Symmetric matrix

34. The general solution of $\frac{d^2y}{dx^2} + y = \csc x$ is :

(A) $y(x) = \sin x \log(\sin x) - x\cos x + A\cos x + B\sin x$

(B) $y(x) = \sin x \log(\sin x) - \cos x + A\cos x + B\sin x$

(C) $y(x) = \sin x \log(\sin x) + x\cos x + A\cos x + B\sin x$

(D) $y(x) = \sin x \log(\cos x) - x\cos x + A\cos x + B\sin x$

35. The inverse Laplace transform of the function $g(s) = \frac{s+2}{(s-1)^2 s^3}$ is :

(A) $(3t-8)e^t + t^2 + 5t + 8$

(B) $(3t+8)e^t + t^2 + 5t - 8$

(C) $(3t-8)e^t + t^2 - 5t + 8$

(D) $(3t-8)e^t - t^2 + 5t + 8$

36. The differential equation of the family of circles of radius r whose center lie on the x axis is :

(A) $y\frac{dy}{dx} + y^2 = r^2$

(B) $y\left\{\frac{dy}{dx} + 1\right\} = r^2$

(C) $y^2\left\{\frac{dy}{dx} + 1\right\} = r^2$

(D) $y^2\left\{\left(\frac{dy}{dx}\right)^2 + 1\right\} = r^2$

37. Linear combinations of solutions of an ordinary differential equation are solutions if the differential equation is :

(A) Linear non-homogeneous

(B) Linear homogeneous

(C) Non-linear homogeneous

(D) Non-linear non-homogeneous

38. The solution of the differential equation, $x \cos x\frac{dy}{dx} + y(x \sin x + \cos x) = 1$ is :

(A) $yx \cot x = \tan x + C$

(B) $yx \sec x = \cot x + C$

(C) $yx \sec x = \tan x + C$

(D) $yx \cos x = \tan x + C$

where C is an arbitrary constant.

39. The solution of the differential equation, $(D^2 + 1)y = 12 \cos^2 x$ (here $D = \frac{d}{dx}$) is :

(A) $y(x) = C_1 \cos x + C_2 \sin x + 6 + 2 \cos 2x$

(B) $y(x) = C_1 \cos x + C_2 \sin x + 5 - 2 \cos 2x$

(C) $y(x) = C_1 \cos x + C_2 \sin x + 6 - 2 \cos 2x$

(D) $y(x) = C_1 \cos x + C_2 \sin x + 6 - 2 \cos 2x$

where C_1 and C_2 are arbitrary constants

40. The number of linearly independent solutions of $\frac{d^4y}{dx^4} - \frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$ of the form $e^{ax} (a \in \mathbb{R})$ is :

(A) One

(B) Two

(C) Three

(D) Four

41. the solution of the differential equation, $(x^3D^3 + 3x^2D^2 - 2xD + 2)y = 0$ (here $D = \frac{d}{dx}$) is :

(A) $y(x) = (C_1 + C_2 \ln x)x - C_3x^{-2}$

(B) $y(x) = (C_1 + C_2 \ln x)x + C_3x^{-2}$

(C) $y(x) = (C_1 + C_2 \ln x)x^2 + C_3x^{-2}$

(D) $y(x) = (C_1 + C_2 \ln x)x^3 + C_3x^{-2}$

where C_1, C_2 and C_3 are arbitrary constants.

42. The particular integral of $(D^2 + 4)y = x \sin x$ (here $D = \frac{d}{dx}$) is :

(A) $\frac{1}{9}(3x \cos x - 2 \sin x)$

(B) $\frac{1}{9}(3x \cos x + 2 \sin x)$

(C) $\frac{1}{9}(3x \sin x - 2 \cos x)$

(D) $\frac{1}{9}(3x \sin x + 2 \cos x)$

43. The integrating factor of the differential equation, $\frac{dy}{dx}(x \ln x) + y = 2 \ln x$ is :

(A) x

(B) e^x

(C) $\ln x$

(D) $\ln(\ln x)$

44. If $A = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $P = ABA^T$, then $A^T(P^{2005})A$ is :

(A) $\begin{pmatrix} 1 & 2005 \\ 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{pmatrix}$

45. If $2x - y = \begin{pmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{pmatrix}$ and $2y + x = \begin{pmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{pmatrix}$ then :

(A) $x + y = \begin{pmatrix} 3 & 0 & -1 \\ 0 & 3 & -2 \end{pmatrix}$

(B) $x = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$

(C) $x - y = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \end{pmatrix}$

(D) $y = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 1 & -2 \end{pmatrix}$

46. The matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & 3 \end{pmatrix}$ is :

(A) Idempotent

(B) Involutory

(C) Orthogonal

(D) Nilpotent

47. If $A^2 + A - I = 0$, then A^{-1} is equal to :

(A) $A - I$

(B) $I - A$

(C) $1 + A$

(D) None of these

48. If $A = \begin{pmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{pmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$, then f(A) equals :

(A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(B) $\left(\frac{3 - i\sqrt{3}}{2}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(C) $\left(\frac{5 - i\sqrt{3}}{2}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(D) $\left(2 + i\sqrt{3}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

49. If $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{pmatrix}$ is an idempotent matrix, then x is equal to :

(A) -5

(B) -1

(C) -3

(D) -4

50. The inverse Laplace transform of $\frac{1}{2}\ln\frac{s^2+b^2}{s^2+a^2}$ is :

(A) $f(t)=\frac{1}{t}(\sin at-\cos bt)$

(B) $f(t)=\frac{1}{t}(\cos bt-\sin at)$

(C) $f(t)=\frac{1}{t}(\cos bt-\cos at)$

(D) $f(t)=\frac{1}{t}(\cos at-\cos bt)$

51. The complete integral of the partial differential equation $q = xyp^2$, where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$, $z=z(x, y)$ is :

(A) $(z-ay^2-2c)^2=6ax$

(B) $(2z-ay^2-2c)^2=6ax$

(C) $(2z-ay^2-2c)^2=16ax$

(D) $(z-ay^2-2c)^2=16ax$

52. The complete integral of the partial differential equation $z=px+qy+p^2+q^2$, where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$, $z=z(x, y)$ is :

(A) $z=ax+by+a^2+b^2$

(B) $z=ax+by+a^3+b^2$

(C) $z=ax+by+a^2+b^3$

(D) $z=x+by+a^2+b^2$

53. The complementary function of $(D^2-D_1^2+2D+1)z=0$, where $D=\frac{\partial}{\partial x}$, $D_1=\frac{\partial}{\partial y}$ is :

(A) $z=e^{-x}\phi_1(x-y)+e^{-x}\phi_2(x+y)$

(B) $z=e^{x}\phi_1(x-y)+e^{x}\phi_2(x+y)$

(C) $z=e^{x}\phi_1(x-y)+e^{-x}\phi_2(x+y)$

(D) $z=e^{-x}\phi_1(x-y)+e^{x}\phi_2(x+y)$

54. The particular integral of the equation $(D^2+2D_1+D_1^2)z=12xy$, where $D=\frac{\partial}{\partial x}$, $D_1=\frac{\partial}{\partial y}$ is :

(A) $2xy^3-x^4$

(B) $2x^4y-x^3$

(C) $2x^3y-x^4$

(D) $2x^2y^3-x^4$

55. The particular integral of the equation $(D^2-3DD_1+2D_1^2)z=e^{2x-y}$, where $D=\frac{\partial}{\partial x}$, $D_1=\frac{\partial}{\partial y}$ is :

(A) $\frac{1}{10}e^{2x-y}$

(B) $\frac{1}{12}e^{2x-y}$

(C) $\frac{1}{16}e^{2x-2}$

(D) $\frac{1}{14}e^{2x-y}$

56. The particular integral of the equation $(D^2 - 2DD_1 + D_1^2)z = \cos(x + 2y)$, where $D = \frac{\partial}{\partial x}$, $D_1 = \frac{\partial}{\partial y}$ is :

(A) $-\frac{1}{9}\cos(x+2y)$

(B) $-\frac{1}{7}\cos(x+2y)$

(C) $-\frac{1}{5}\cos(x+2y)$

(D) $-\frac{1}{3}\cos(x+2y)$

57. The particular integral of the equation $(D^2 - D_1)z = x \sin y$, where $D = \frac{\partial}{\partial x}$, $D_1 = \frac{\partial}{\partial y}$ is :

(A) $y \sin x$

(B) $y \cos x$

(C) $x \cos y$

(D) $x \sin x$

58. The partial differential equation $xyr - (x^2 - y^2)s - xyt + py - qx = 2(x^2 - y^2)$ is :

(A) Hyperbolic

(B) Elliptic

(C) Parabolic

(D) None of these

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$.

59. The power series solution of the initial value problem $x(2 - x)y'' - 6(x - 1)y' - 4y = 0$; $y(1) = 1, y'(1) = 0$ is :

(A) $y = \sum_{n=0}^{\infty}(n+2)(x-1)^{2n}$

(B) $y = \sum_{n=0}^{\infty}(n+1)(x-1)^{2n}$

(C) $y = \sum_{n=0}^{\infty}(n+2)(x+1)^{2n}$

(D) $y = \sum_{n=0}^{\infty}(n+1)(x+1)^{2n}$

where $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$.

60. The value of Bessel function $J_{1/2}(x)$ is :

(A) $\sqrt{\frac{2}{\pi x}}\sin x$

(B) $\sqrt{\frac{2}{\pi x}}\cos x$

(C) $\sqrt{\frac{2}{\pi x}}(\sin x + \cos x)$

(D) $\sqrt{\frac{2}{\pi x}}(\cos x - \sin x)$

61. If U and V are two subspaces of a finite-dimensional vecotor space W then :

(A) dim (U + V) = dim (U) + dim (V) + dim (U ∩ V)

(B) dim (U + V) = dim (U) – dim (V) + dim (U ∩ V)

(C) dim (U + V) = dim (U) – dim (V) – dim (U ∩ V)

(D) dim (U + V) = dim (U) + dim (V) – dim (U ∩ V)

62. Let T : $R^3 \to R^3$ be the linear transformation defined by T(x, y, z) = (2x, 4y, 5z). The matrix of T with respect to the basis {(2/3, 0, 0), (0,1/2, 0), (0, 0, 1 / 4)} is :

(A) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

63. The dimension of the vector space formed by all n × n symmetric matrices over F is :

(A) $\frac{n(n+1)}{2}$

(B) $\frac{n(n-1)}{2}$

(C) $\frac{n}{2}$

(D) $\frac{n}{2}+1$

64. The series $\sum \frac{\sin(x^2+n^2x)}{n(n+1)}$:

(A) Diverges

(B) Uniformly converges for all real x

(C) Uniformly converges for some values of x

(D) None of these

65. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n+x^2}$ is :

(A) Divergent

(B) Absolutely convergent for all real x

(C) Uniformly convergent for all real x

(D) Absolutely and uniformly convergent for all real x

66. The sequence $\frac{x}{1+nx^2}$:

(A) Converges uniformly to a function f on [0, 1]

(B) Converges absolutely to a function f on [0, 1]

(C) Does not converge

(D) Point wise converges

67. If f is continuous on [0, 1] and if $\int_0^1 x^n f(x)dx = 0$, for n = 0, 1, 2,......

then :

(A) f(x) = 0, on [0, 1]

(B) f′(x) = 0, on [0, 1]

(C) f(x) = k; (k ≠ 0) on [0, 1]

(D) f(0) = f(1) = 0 and f ≠ 0 for x ∈ (0, 1)

68. Let A = {0, 1, 2, 3, ..., 100} then the measure of the set A is :

(A) 101

(B) Infinite

(C) Zero

(D) Not defined

69. The value of Lebsegue integral $\int_a^b f(x)dx$ where f(x) is defined by f(x) = 1, when x is rational and f(x) = 2, where x is irrational, is :

(A) ∞

(B) 2(b – a)

(C) b – a

(D) None of these

70. Let $f(x) = \frac{1}{x}$, if $0 < x \le 1$, and f(x) = 9, if x = 0, then :

(A) f is Lebesgue integrable on [0, 1]

(B) f is not Lebesgue integrable on [0, 1]

(C) f is differentiable at x = 0

(D) (B) and (C) are true

71. Let f and g be Lebesgue integrable function on [0, 1], and let F and G be the integrals $F(x) = \int_0^x f(t)\,dt$, $G(x) = \int_0^x g(t)dt$, then :

(A) $\int_0^1 F(x)g(x)dx = F(1)G(1) - \int_0^1 f(x)\,G(x)\,dx$

(B) $\int_0^1 F(x)g(x)dx = F(1)G(0) - \int_0^1 f(x)\,G(x)\,dx$

(C) $\int_0^1 F(x)g(x)dx = F(0)G(1) - \int_0^1 f(x)\,G(x)\,dx$

(D) $\int_0^1 F(x)g(x)dx = F(0)G(0) - \int_0^1 f(x)\,G(x)\,dx$

72. Which of the following statement is the negation of the statement, "2 is even and 3 is negative" ?

(A) 2 is even and 3 is not negative

(B) 2 is odd and 3 is not negative

(C) 2 is even or 3 is not negative

(D) 2 is odd or 3 is not negative

73. A partial ordered relation is transitive, reflexive and :

(A) Antisymmetric

(B) Bisymmetric

(C) Antireflexive

(D) Asymmetric

74. Let N = {1, 2, 3, ...} be ordered by divisibility, which of the following subset is totally ordered ?

(A) (2, 6, 24)

(B) (3, 5, 15)

(C) (2, 9, 16)

(D) (4, 15, 30)

75. If B is a Boolean Algebra, then which of the following is true ?

(A) B is a finite but not complemented lattice

(B) B is a finite, complemented and distributive lattice

(C) B is a finite, distributive but not complemented lattice

(D) B is not distributive lattice

76. A partially ordered set is said to be a lattice if every two elements in the set have :

(A) A unique least upper bound

(B) A unique greatest lower bound

(C) Both (A) and (B)

(D) None of the above

77. Set A has 3 elements and set B has 4 elements then number of injections defined from A to B are :

(A) 12

(B) 24

(C) 36

(D) 48

78. A function is defined by mapping $f : A \to B$ such that A contains m elements and B contains n elements and m > n then number of bijections are :

(A) $C(n, m) \times m!$

(B) $C(n, m) \times n!$

(C) 0

(D) None of these

where $C(n, m) = \frac{n!}{(n-m)!m!}$

79. If x and y are positive numbers both are less than one, then maximum value of $\lfloor (x + y)$ is :

(A) 0

(B) 1

(C) 2

(D) –1

80. Following are the two statements :

(i) Two finite-dimensional vector spaces over the same field are isomorphic

(ii) Two finite-dimensional vector spaces over the same field and of the same dimension are isomorphic

Then :

(A) (i) is true but (ii) is not true

(B) (ii) is true but (i) is not true

(C) None of these are true

(D) All of these are true

81. Following are three statements :

(i) Any n-dimensional real vector space is isomorphic to R^n

(ii) Any n-dimensional complex vector space is isomorphic to C^n

(iii) Any n-dimensional vector space over the field F is isomorphic to F^n

Then :

(A) Only (i) and (ii) are true

(B) (i) is true, but (ii) and (iii) are not true

(C) None of these are true

(D) All of these are true

82. Let H be a separable infinite-dimensional (complex) Hilbert space. Then which of the following is true ?

(A) Every orthonomal set must be countable

(B) Every orthonormal set in H is contained in a (countable) maximal orthonormal set. In particular, there exists a (countable) maximal orthonormal set

(C) If $\{\phi_1, \phi_2, ...\}$ is an orthonormal sequence in H, and $\{c_1, c_2, ...\}$ is a square summable sequence of complex numbes, then the infinite series $\sum c_n\phi_1$ converges to an element in H

(D) All of these

83. The series $\sum_{n=1}^{\infty}\left(xe^{-x}\right)^n$:

(A) Converges uniformly on [0, 2]

(B) Continuous on [0, 2]

(C) Differentiable on [0, 2]

(D) All of these are true

84. Let f_n be a sequence of functions in R[a, b] converging uniformly to f. Then $f \in R[a, b]$ and :

(A) $\text{Lim}_{n\to\infty}\int_a^b f_n(x)dx = \int_a^b f(x)\,dx$

(B) $\text{Lim}_{n\to\infty}\int_a^b f_n(x)dx > \int_a^b f(x)\,dx$

(C) $\text{Lim}_{n\to\infty}\int_a^b f_n(x)dx < \int_a^b f(x)\,dx$

(D) $\text{Lim}_{n\to\infty}\int_a^b f_n(x)dx \neq \int_a^b f(x)\,dx$

85. Let f_n be a sequence of continuous functions on $E \subset C$ converging uniformly to f on E. Then :

(A) f is differentiable on E

(B) f is continuous on E

(C) f is uniformly continuous on E

(D) None of these

86. The sequence of functions $f_n(x) = n^2x^n(1-x), x \in [0, 1]$:

(A) Uniformly converges on [0, 1]

(B) Converges pointwise on [0, 1]

(C) Both (A) and (B) are true

(D) None of these

87. If V_1, V_2 and V_3 are subspaces of vector space V then :

(A) $V_3 \bigcap (V_1 + V_2) \supset (V_3 \bigcap V_1) + (V_3 \bigcap V_2)$

(B) $V_3 \bigcap (V_1 + V_2) \subset (V_3 \bigcap V_1) + (V_3 \bigcap V_2)$

(C) $V_3 \bigcap (V_1 + V_2) \subseteq (V_3 \bigcap V_1) + (V_3 \bigcap V_2)$

(D) $V_3 \bigcap (V_1 + V_2) \supseteq (V_3 \bigcap V_1) + (V_3 \bigcap V_2)$

88. The subspace generated by {(1, 0, 0), (0, 2, 0)} in R^3 is :

(A) The xy – plane and z = 0

(B) The yz – plane and x = 0

(C) The zx – plane and y = 0

(D) R^3 itself

89. The condition under which (x_1, x_2, x_3) lies in the subspace generated by (2, 3, 1) and (0, 1, 2) in R^3 is :

(A) $x_1 - x_2 + x_3 = 0$

(B) $5x_1 - 4x_2 + 2x_3 = 0$

(C) $x_1 - 4x_2 + 2x_3 = 0$

(D) $5x_1 - x_2 + 2x_3 = 0$

90. Let V be a vector space over F such that the only subspaces of V are {0} and V then :

(A) V must consists of scalar multiples of one single non-zero vector

(B) V has only zero vectors

(C) V has only unit vectors

(D) None of these

91. Let $V = R^3$ and let $v_1 = (1, 0, 2)$ and $v_2 = (0, 3, 1)$. Then a vector which is orthogonal to both v_1 and v_2 is :

(A) $(-2, -1/3, 1)$

(B) $(0, 1, 2)$

(C) $(-4, -1/3, 3)$

(D) $(-2, 1/3, 1)$

92. In a Hermitian matrix all the diagonal elements are :

(A) 0

(B) Real

(C) Pure imaginary

(D) ± 1 only

93. Which of the following is not correct ?

(A) The row rank and column rank of a matrix A are the same

(B) Every matrix A is row equivalent to a row reduced echelon matrix

(C) If a matrix is in the row reduced echelon form, its row rank is the number of non zero rows in it

(D) An $n \times n$ matrix A is non-singular if and only if its row reduced echelon form is A itself

94. The determinant of an idempotent matrix is :

(A) 0 only

(B) 1 only

(C) Either 0 or 1

(D) Only ± 1

95. If $P_n(x)$ is the Legendre polynomial then which of the following is not true ?

(A) $P_n(1) = 1$

(B) $P_n(-1) = (-1)^n$

(C) $P_n'(1) = \frac{n(n+1)}{2}$

(D) $P_n'(-1) = (-1)^n + \frac{n(n+1)}{2}$

96. Let $P_n(x)$ be the Legendre polynomial then the value of $\int_{-1}^{1} [P_n(x)]^2 dx$ is :

(A) $\frac{2}{2n+1}$

(B) $\frac{2}{2n-1}$

(C) $\frac{1}{2n+1}$

(D) $\frac{1}{2n-1}$

97. If $P_n(x)$ is the Legendre polynomial then which of the following is not true ?

(A) $(2n+1)P_n = P'_{n+1} - P'_{n-1}$

(B) $nP_n = xP'_n - P'_{n-1}$

(C) $(1+n)P'_n = P'_{n+1} - xP'_n$

(D) $(1 - x^2)P'_n = (n + 1)(xP_n - P_{n+1})$

98. The complete integral of $z^2 = pqxy$ is :

(A) $z = x^a y^{1/a} C$

(B) $z = x^{1/a} y^{1/b} C$

(C) $z = x^2 y^b C$

(D) $z = x^a y^2 C$

99. The complete integral of $p^2x^2 + px = q$ (where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$) is :

(A) $z = a \log x + (a^2 + a)x + c$

(B) $z = a \log x + (a^2 + a)y^2 + c$

(C) $z = a \log x + (a^2 + a)x^2 + c$

(D) $z = a \log x + (a^2 + a)y + c$

100. For the differential equation $2x^2\frac{d^2y}{dx^2} + 7x(x + 1)\frac{dy}{dx} - 3y = 0$, the point $x = 0$ is :

(A) Ordinary point

(B) Singular point

(C) Regular singular point

(D) Irregular singular point

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